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From the archives of the CEMC
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In honour of the 50th anniversary of the Faculty of Mathematics, at the beginning of each month of 2017, a set of five problems from the 54 years of CEMC contests will be posted. Solutions to the problems will be posted at the beginning of the next month. Hopefully, these problems will intrigue and inspire your mathematical mind. For more problem solving resources, please visit `cemc.uwaterloo.ca`.

1. *1985 Euclid Contest, Question A6*
   L and M are fixed points, 5 cm apart. The set of all points P in the plane, for which the triangle LMP has an area of 20 cm^2, is

   (A) a circle, diameter LM

   (B) a pair of lines perpendicular to LM

   (C) one line perpendicular to LM

   (D) a pair of lines parallel to LM

   (E) one line parallel to LM

2. *1968 Ontario Senior Mathematics Problems Contest, Question 2*

   (a) Given that the equation
   $$x^3 + ax^2 + bx + c = 0$$
   has roots r_1, r_2, r_3, develop formulae for $r_1 + r_2 + r_3, r_1r_2 + r_1r_3 + r_2r_3$, and $r_1r_2r_3$.
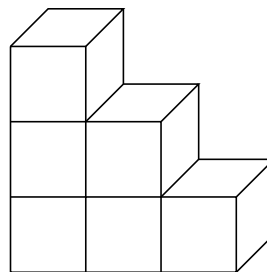
   (b) For the equation
   $$x^3 + 2x^2 + 7x - 19 = 0$$
   compute the values of $r_1^2 + r_2^2 + r_3^2$ and $r_1^3 + r_2^3 + r_3^3$.

3. *1981 Gauss Contest, Question 22*
   A base row of blocks is formed and rows of blocks are added so that each new row has one fewer block that the row below it. If the base has nine blocks and the final row has one block, the total number of blocks used is

   (A) 6   **(B)** 36   **(C)** 40

   (D) 45   **(E)** 81

4. *1986 Descartes Contest, Question 9*
   In $\triangle ABC$, $ab^2 \cos A = bc^2 \cos B = ca^2 \cos C$. Prove that the triangle is equilateral.

5. *1977 Gauss Contest, Question 10*
   The maximum number of points of intersection of 4 distinct straight lines is

   (A) 4   **(B)** 5   **(C)** 6   **(D)** 7   **(E)** none of these