The CENTRE for EDUCATION in MATHEMATICS and COMPUTING

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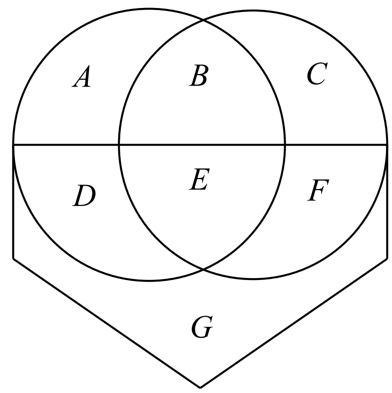
From the archives of the CEMC

July 2017

In honour of the 50th anniversary of the Faculty of Mathematics, at the beginning of each month of 2017, a set of five problems from the 54 years of CEMC contests will be posted. Solutions to the problems will be posted at the beginning of the next month. Hopefully, these problems will intrigue and inspire your mathematical mind. For more problem solving resources, please visit `cemc.uwaterloo.ca`.

1. *1989 Pascal Contest, Question 15*
   Two circles, each with a radius of one, and a pentagon overlap as shown in the diagram. The bounded regions labelled A, B, C, D, E, F, and G are of equal area. The area of the pentagon is

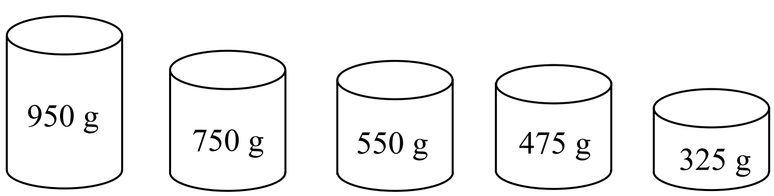
   (A) 1 **(B)** $\frac{4}{7}$ **(C)** $\frac{\pi}{4}$
   (D) π **(E)** 2π



2. *2014 Fermat Contest, Question 22*
   Jillian drives along a straight road that goes directly from her house (J) to her Grandfather's house (G). Some of this road is on flat ground and some is downhill or uphill. Her car travels downhill at 99 km/h, on flat ground at 77 km/h, and uphill at 63 km/h. It takes Jillian 3 hours and 40 minutes to drive from J to G. It takes her 4 hours and 20 minutes to drive from G to J. The distance between J and G, in km, is

   (A) $318\frac{2}{3}$ **(B)** 324 **(C)** 308 **(D)** $292\frac{3}{5}$ **(E)** $292\frac{1}{9}$

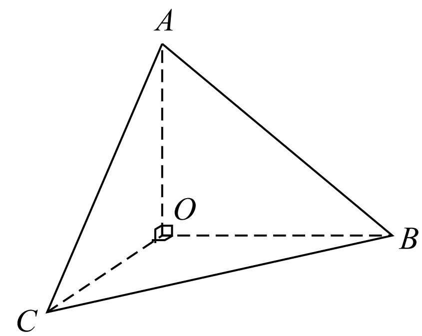
3. *1995 Gauss Contest, Grade 7, Question 24*
   The five tins shown contain either coffee or cocoa or powdered milk.



   There is twice as much coffee as cocoa by total weight. No three tins contain the same item. The only tin containing cocoa is the

   (A) 950 g tin **(B)** 750 g tin **(C)** 550 g tin **(D)** 475 g tin **(E)** 325 g tin

4. *1993 Grade 11 Invitational Mathematics Challenge, Question 5*
In the tetrahedron shown in the diagram, angles COA, AOB and COB are right angles. The three triangles meeting at O have areas of 6, $\sqrt{39}$, and 5 units. Determine the area of $\triangle ABC$.



5. *2017 Fryer Contest, Question 2*
By finding a common denominator, we see that $\frac{1}{3}$ is greater than $\frac{1}{7}$ because $\frac{7}{21} > \frac{3}{21}$.
Similarly, we see that $\frac{1}{3}$ is less than $\frac{1}{2}$ because $\frac{2}{6} < \frac{3}{6}$.

(a) Determine the integer n so that $\frac{n}{40}$ is greater than $\frac{1}{5}$ and less than $\frac{1}{4}$.

(b) Determine all possible integers m so that $\frac{m}{8}$ is greater than $\frac{1}{3}$ and $\frac{m+1}{8}$ is less than $\frac{2}{3}$.

(c) Fiona calculates her *win ratio* by dividing the number of games that she has won by the total number of games that she has played. At the start of a weekend, Fiona has played 30 games, has w wins, and her win ratio is greater than 0.5. During the weekend, she plays five games and wins three of these games. At the end of the weekend, Fiona's win ratio is less than 0.7. Determine all possible values of w.