The CENTRE for EDUCATION in MATHEMATICS and COMPUTING

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From the archives of the CEMC

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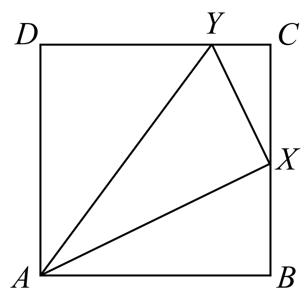
In honour of the 50th anniversary of the Faculty of Mathematics, at the beginning of each month of 2017, a set of five problems from the 54 years of CEMC contests will be posted. Solutions to the problems will be posted at the beginning of the next month. Hopefully, these problems will intrigue and inspire your mathematical mind. For more problem solving resources, please visit `cemc.uwaterloo.ca`.

1. *2001 Grade 11 Invitational Mathematics Challenge, Question 3*
   Points X and Y are on sides BC and CD of square $ABCD$, as shown. The lengths of XY, AX and AY are 3, 4 and 5, respectively. Determine the side length of square $ABCD$.



2. *2015 Canadian Intermediate Mathematics Contest, Question B2*
   Alistair, Conrad, Emma, and Salma compete in a three-sport race. They each swim 2 km, then bike 40 km, and finally run 10 km. Also, they each switch instantly from swimming to biking and from biking to running.

   (a) Emma has completed $\frac{1}{13}$ of the total distance of the race. How many kilometers has she travelled?

   (b) Conrad began the race at 8:00 a.m. and completed the swimming portion in 30 minutes. Conrad biked 12 times as fast as he swam, and ran 3 times as fast as he swam. At what time did he finish the race?

   (c) Alistair and Salma also began the race at 8:00 a.m. Alistair finished the swimming portion in 36 minutes, and then biked at 28 km/h. Salma finished the swimming portion in 30 minutes, and then biked at 24 km/h. Alistair passed Salma during the bike portion. At what time did Alistair pass Salma?

3. *1991 Fermat Contest, Question 21*
   A hardware store sells single digits to be used for house numbers. There are five 5s, four 4s, three 3s, and two 2s available. From this selection of digits, a customer is able to purchase his three-digit house number. The number of possible house numbers this customer could have is

   (A) 63 **(B)** 24 **(C)** 60 **(D)** 48 **(E)** 39

4. *1996 Pascal Contest, Question 25*
   There are exactly k perfect squares which are divisors of 1996^{1996}. The sum of the digits in the number k is

   (A) 29 **(B)** 26 **(C)** 30 **(D)** 22 **(E)** 27

5. *2017 Euclid Contest Question, Question 4b*
   In an arithmetic sequence with 5 terms, the sum of the squares of the first 3 terms equals the sum of the squares of the last 2 terms. If the first term is 5, determine all possible values of the fifth term.

   (An *arithmetic sequence* is a sequence in which each term after the first is obtained from the previous term by adding a constant. For example, $3, 5, 7, 9, 11$ is an arithmetic sequence with five terms.)