The CENTRE for EDUCATION
in MATHEMATICS and COMPUTING
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2022 Pascal Contest

(Grade 9)

Wednesday, February 23, 2022
(in North America and South America)

Thursday, February 24, 2022
(outside of North America and South America)

Solutions

1. Evaluating, $\dfrac{20+22}{2} = \dfrac{42}{2} = 21$.

   ANSWER: (D)

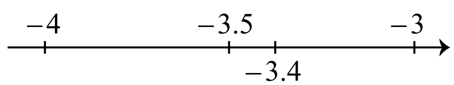
2. From the graph, we see that Haofei donated \$2, Mike donated \$6, Pierre donated \$2, and Ritika donated \$8.
   In total, the four students donated $\$2 + \$6 + \$2 + \$8 = \$18$.

   ANSWER: (B)

3. In the given sum, each of the four fractions is equivalent to $\frac{1}{2}$.
   Therefore, the given sum is equal to $\frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} = 2$.

   ANSWER: (E)

4. On a number line, -3.4 is between -4 and -3.
   This means that -3.4 is closer to -4 and -3 than to any of 0, 3 or 4, and so the answer must be -4 or -3.
   If we start at -3 and move in the negative direction (that is, to the left), we reach -3.4 after moving 0.4 units.
   It then takes an additional 0.6 units to move in the negative direction from -3.4 to -4.
   Therefore, -3.4 is closer to -3 than to -4, and so the answer is (B) or -3.
   Alternatively, when comparing -3, -4 and -3.4, we could note that -3.4 is between -3.5 and -3 and so is closer to -3:



   ANSWER: (B)

5. From the diagram, $PR = 10 - 3 = 7$ and $QS = 17 - 5 = 12$ and so $PR : QS = 7 : 12$.

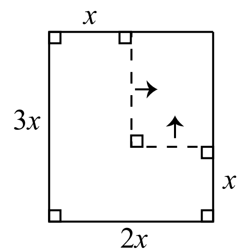
   ANSWER: (A)

6. Between them, Robyn and Sasha have $4 + 14 = 18$ tasks to do.
   If each does the same number of tasks, each must do $18 \div 2 = 9$ tasks.
   This means that Robyn must do $9 - 4 = 5$ of Sasha's tasks.

   ANSWER: (C)

7. Because all of the angles in the figure are right angles, each line segment is either horizontal or vertical.
   The height of the figure is $3x$ and the width of the figure is $2x$.
   This means that the length of the unmarked vertical segment must equal $3x - x = 2x$.
   Also, the length of the unmarked horizontal segment must equal $2x - x = x$.
   Starting in the top left corner and adding lengths in a clockwise direction, the perimeter is $x + 2x + x + x + 2x + 3x = 10x$.

Alternatively, we can "complete the rectangle" by sliding the shortest horizontal side and the shortest vertical side as shown to form a rectangle with height $3x$ and width $2x$:



The perimeter of this rectangle is $2 \times 2x + 2 \times 3x = 10x$.

ANSWER: (E)

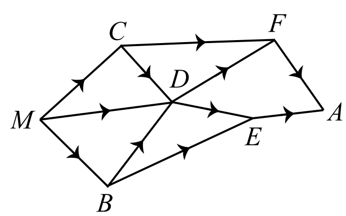
8. The total central angle in a circle is 360°.
Since the Green section has an angle at the centre of the circle of 90°, this section corresponds to $\frac{90^\circ}{360^\circ} = \frac{1}{4}$ of the circle.
This means that when the spinner is spun once, the probability that it lands on the Green section is $\frac{1}{4}$.
Similarily, the probability that the spinner lands on Blue is also $\frac{1}{4}$.
Since the spinner lands on one of the four colours, the probability that the spinner lands on either Red or Yellow is $1 - \frac{1}{4} - \frac{1}{4} = \frac{1}{2}$.

ANSWER: (D)

9. Since the line with equation $y = 2x + b$ passes through the point $(-4, 0)$, the coordinates of the point must satisfy the equation of the line.
Substituting $x = -4$ and $y = 0$ gives $0 = 2(-4) + b$ and so $0 = -8 + b$ which gives $b = 8$.

ANSWER: (E)

10. We label Mathville as M, Algebratown as A, and the other intersection points of roads as shown.



There is 1 route from M to each of C and B: $M \to C$ and $M \to B$.
There are 3 routes to D: $M \to D$ and $M \to C \to D$ and $M \to B \to D$.
This means that there are 4 routes to F:

$$M \to C \to F \qquad M \to D \to F \qquad M \to C \to D \to F \qquad M \to B \to D \to F$$

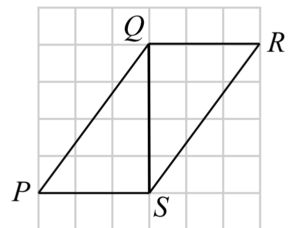
Similarly, there are 4 routes to E:

$$M \to B \to E \qquad M \to D \to E \qquad M \to C \to D \to E \qquad M \to B \to D \to E$$

Finally, there are $4 + 4 = 8$ routes to A, since every route comes through either E or F, no route goes through both E and F, and there are 4 routes to each of E and F.

ANSWER: (C)

11. Since the given grid is 6×6, the size of each of the small squares is 1×1.
This means that $QR = PS = 3$.
Join Q to S.



Since QS is vertical, and QR and PS are both horizontal, then $\angle RQS = 90^\circ$ and $\angle PSQ = 90^\circ$.
We note further that $QS = 4$.
Since $\triangle RQS$ is right-angled at Q, by the Pythagorean Theorem,

$$RS^2 = QR^2 + QS^2 = 3^2 + 4^2 = 25$$

Since $RS > 0$, then $RS = 5$.
Similarly, $PQ = 5$.
Thus, the perimeter of $PQRS$ is $PQ + QR + RS + PS = 5 + 3 + 5 + 3 = 16$.

Answer: (C)

12. The integers between 1 and 100 that have a ones digit equal to 6 are

$$6, 16, 26, 36, 46, 56, 66, 76, 86, 96$$

of which there are 10.
The additional integers between 1 and 100 that have a tens digits equal to 6 are

$$60, 61, 62, 63, 64, 65, 67, 68, 69$$

of which there are 9. (Note that 66 was included in the first list and not in the second list since we are counting integers rather than total number of 6s).
Since the digit 6 must occur as either the ones digit or the tens digit, there are $10 + 9 = 19$ integers between 1 and 100 with at least 1 digit equal to 6.

Answer: (C)

13. Suppose that Rosie runs x metres from the time that they start running until the time that they meet.
Since Mayar runs twice as fast as Rosie, then Mayar runs $2x$ metres in this time.
When Mayar and Rosie meet, they will have run a total of 90 m, since between the two of them, they have covered the full 90 m.
Therefore, $2x + x = 90$ and so $3x = 90$ or $x = 30$.
Since $2x = 60$, this means that Mayar has run 60 m when they meet.

Answer: (D)

14. We use A, B, C, D, and E to represent Andy, Bev, Cao, Dhruv, and Elcim, respectively. We use the notation $D > B$ to represent the fact "Dhruv is older than Bev". The five sentences give $D > B$ and $B > E$ and $A > E$ and $B > A$ and $C > B$. These show us that Dhruv and Cao are older than Bev, and Elcim and Andy are younger than Bev. This means that two people are older than Bev and two people are younger than Bev, which means that Bev must be the third oldest.

    Answer: (B)

15. We note that all of the given possible sums are odd, and also that every prime number is odd with the exception of 2 (which is even).
    When two odd integers are added, their sum is even.
    When two even integers are added, their sum is even.
    When one even integer and one odd integer are added, their sum is odd.
    Therefore, if the sum of two integers is odd, it must be the sum of an even integer and an odd integer.
    Since the only even prime number is 2, then for an odd integer to be the sum of two prime numbers, it must be the sum of 2 and another prime number.
    Note that

    $$19 = 2 + 17 \qquad 21 = 2 + 19 \qquad 23 = 2 + 21 \qquad 25 = 2 + 23 \qquad 27 = 2 + 25$$

    Since 17, 19 and 23 are prime numbers and 21 and 25 are not prime numbers, then 3 of the given integers are the sum of two prime numbers.

    Answer: (A)

16. Since 60 games are played and each of the 3 pairs plays the same number of games, each pair plays $60 \div 3 = 20$ games.
    Alvin wins 20% of the 20 games that Alvin and Bingyi play, so Alvin wins $\frac{20}{100} \times 20 = \frac{1}{5} \times 20 = 4$ of these 20 games and Bingyi wins $20 - 4 = 16$ of these 20 games.
    Bingyi wins 60% of the 20 games that Bingyi and Cheska play, so Bingyi wins a total of $\frac{60}{100} \times 20 = \frac{3}{5} \times 20 = 12$ of these 20 games.
    The games played by Cheska and Alvin do not affect Bingyi's total number of wins.
    In total, Bingyi wins $16 + 12 = 28$ games.

    Answer: (C)

17. Since $a + 5 = b$, then $a = b - 5$.
    Substituting $a = b - 5$ and $c = 5 + b$ into $b + c = a$, we obtain

    $$\begin{aligned} b + (5 + b) &= b - 5 \\ 2b + 5 &= b - 5 \\ b &= -10 \end{aligned}$$

    (If $b = -10$, then $a = b - 5 = -15$ and $c = 5 + b = -5$ and $b + c = (-10) + (-5) = (-15) = a$, as required.)

    Answer: (C)

18. Starting with the balls in the order `1 2 3 4 5`, we make a table of the positions of the balls after each of the first 10 steps:

Step	Ball that moves	Order after step
1	Rightmost	1 2 5 3 4
2	Leftmost	2 5 1 3 4
3	Rightmost	2 5 4 1 3
4	Leftmost	5 4 2 1 3
5	Rightmost	5 4 3 2 1
6	Leftmost	4 3 5 2 1
7	Rightmost	4 3 1 5 2
8	Leftmost	3 1 4 5 2
9	Rightmost	3 1 2 4 5
10	Leftmost	1 2 3 4 5

After 10 steps, the balls are in the same order as at the beginning. This means that after each successive set of 10 steps, the balls will be returned to their original order.
Since 2020 is a multiple of 10, then after 2020 steps, the balls will be in their original order.
Steps 2021 through 2025 will repeat the outcomes of steps 1 through 5 above, and so after 2025 steps, the balls will be in the reverse of their original order.
Therefore, 2025 is a possible value of N. This argument can be adapted to check that none of 2028, 2031 and 2027 are possible values of N.

ANSWER: (E)

19. The six-digit integer that Miyuki sent included the digits 2022 in that order along with two 3s. If the two 3s were consecutive digits, there are 5 possible integers:

332022 233022 203322 202332 202233

We can think about the pair of 3s being moved from left to right through the integer.
If the two 3s are not consecutive digits, there are 10 possible pairs of locations for the 3s: 1st/3rd, 1st/4th, 1st/5th, 1st/6th, 2nd/4th, 2nd/5th, 2nd/6th, 3rd/5th, 3rd/6th, 4th/6th.
These give the following integers:

323022 320322 320232 320223 230322 230232 230223 203232 203223 202323

(We can think about moving the leftmost 3 from left to right through the integer and finding all of the possible locations for the second 3.)
In total, there are thus $5 + 10 = 15$ possible six-digit integers that Miyuki could have texted.

ANSWER: (E)

20. *Solution 1*

Each of the n friends is to receive $\frac{1}{n}$ of the pizza.

Since there are two pieces that are each $\frac{1}{6}$ of the pizza and these pieces cannot be cut, then each friend receives at least $\frac{1}{6}$ of the pizza. This means that there cannot be more than 6 friends; that is, $n \leq 6$.

Therefore, $n = 7, 8, 9, 10$ are not possible. The sum of these is 34.

The value $n = 2$ is possible. We show this by showing that the pieces can be divided into two groups, each of which totals $\frac{1}{2}$ of the pizza.

Note that $\frac{1}{6} + \frac{1}{6} + \frac{1}{12} + \frac{1}{12} = \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{3}{6} = \frac{1}{2}$.

This also means that the other 6 pieces must also add to $\frac{1}{2}$.

We show that the value of $n = 3$ is possible by finding 3 groups of pieces, with each group totalling $\frac{1}{3}$ of the pizza.

Since $2 \times \frac{1}{6} = \frac{1}{3}$ and $4 \times \frac{1}{12} = \frac{1}{3}$, then the other 4 pieces must also add to $\frac{1}{3}$ (the rest of the pizza) and so $n = 3$ is possible.

The value $n = 4$ is possible since $2 \times \frac{1}{8} = \frac{1}{4}$ and $\frac{1}{6} + \frac{1}{12} = \frac{2}{12} + \frac{1}{12} = \frac{3}{12} = \frac{1}{4}$ (which can be done twice). The other 4 pieces must also add to $\frac{1}{4}$.

The value $n = 6$ is possible since two pieces are $\frac{1}{6}$ on their own, two groups of size $\frac{1}{6}$ can be made from the four pieces of size $\frac{1}{12}$, and $\frac{1}{8} + \frac{1}{24} = \frac{3}{24} + \frac{1}{24} = \frac{4}{24} = \frac{1}{6}$ (which can be done twice), which makes 6 groups of size $\frac{1}{6}$.

The sum of the values of n that are not possible is either 34 (if $n = 5$ is possible) or 39 (if $n = 5$ is not possible). Since 34 is not one of the choices, the answer must be 39.

(We can see that $n = 5$ is not possible since to make a portion of size $\frac{1}{5}$ that includes a piece of size $\frac{1}{6}$, the remaining pieces must total $\frac{1}{5} - \frac{1}{6} = \frac{6}{30} - \frac{5}{30} = \frac{1}{30}$. Since every piece is larger than $\frac{1}{30}$, this is not possible.)

Solution 2

The pizza is cut into 2 pieces of size $\frac{1}{24}$, 4 of $\frac{1}{12}$, 2 of $\frac{1}{8}$, and 2 of $\frac{1}{6}$.

Each of these fractions can be written with a denominator of 24, so we can think of having 2 pieces of size $\frac{1}{24}$, 4 of $\frac{2}{24}$, 2 of $\frac{3}{24}$, and 2 of $\frac{4}{24}$.

To create groups of pieces of equal total size, we can now consider combining the integers 1, 1, 2, 2, 2, 2, 3, 3, 4, and 4 into groups of equal size. (These integers represent the size of each piece measured in units of $\frac{1}{24}$ of the pizza.)

Since the largest integer in the list is 4, then each group has to have size at least 4.

Since $4 = 24 \div 6$, then the slices cannot be broken into more than 6 groups of equal size, which

means that $n = 7, 8, 9, 10$ are not possible.
Here is a way of breaking the slices into $n = 6$ equal groups, each with total size $24 \div 6 = 4$:

$$4 \qquad 4 \qquad 3+1 \qquad 3+1 \qquad 2+2 \qquad 2+2$$

Here is a way of breaking the slices into $n = 4$ equal groups, each with total size $24 \div 4 = 6$:

$$4+2 \qquad 4+2 \qquad 3+3 \qquad 2+2+1+1$$

Here is a way of breaking the slices into $n = 3$ equal groups, each with total size $24 \div 3 = 8$:

$$4+4 \qquad 2+2+2+2 \qquad 3+3+1+1$$

Here is a way of breaking the slices into $n = 2$ equal groups, each with total size $24 \div 2 = 12$:

$$4+4+2+2 \qquad 3+3+2+2+1+1$$

Since 24 is not a multiple of 5, the pieces cannot be broken into 5 groups of equal size.
Therefore, the sum of the values of n that are not possible is $5 + 7 + 8 + 9 + 10 = 39$.

ANSWER: (D)

21. A 10 cm by 10 cm board has 9 rows of 9 holes, or $9 \times 9 = 81$ pegs in total.
Each hole on the 2 main diagonals has a peg in it.
There are 9 holes on each diagonal, with the centre hole on both diagonals, since there is an odd number of holes in each row.
Therefore, the total number of holes on the two diagonals is $9 + 9 - 1 = 17$.
This means that the number of empty holes is $81 - 17 = 64$.

ANSWER: 64

22. We start by looking for patterns in the rightmost two digits of powers of 4, powers of 5 and powers of 7.
The first few powers of 5 are

$$5^1 = 5 \qquad 5^2 = \mathbf{25} \qquad 5^3 = 1\mathbf{25} \qquad 5^4 = 6\mathbf{25} \qquad 5^5 = 31\mathbf{25}$$

It appears that, starting with 5^2, the rightmost two digits of powers of 5 are always 25.
To see this, we want to understand why if the rightmost two digits of a power of 5 are 25, then the rightmost two digits of the next power of 5 are also 25.
The rightmost two digits of a power of 5 are completely determined by the rightmost two digits of the previous power, since in the process of multiplication, any digits before the rightmost two digits do not affect the rightmost two digits of the product.
This means that the rightmost two digits of every power of 5 starting with 5^2 are 25, which means that the rightmost two digits of 5^{129} are 25.

The first few powers of 4 are

$$4^1 = 4 \qquad 4^2 = \mathbf{16} \qquad 4^3 = \mathbf{64} \qquad 4^4 = 2\mathbf{56} \qquad 4^5 = 10\mathbf{24} \qquad 4^6 = 40\mathbf{96} \qquad 4^7 = 16\,3\mathbf{84}$$

$$4^8 = 65\,5\mathbf{36} \qquad 4^9 = 262\,1\mathbf{44} \qquad 4^{10} = 1\,048\,5\mathbf{76} \qquad 4^{11} = 4\,194\,3\mathbf{04} \qquad 4^{12} = 16\,777\,2\mathbf{16}$$

We note that the rightmost two digits repeat after 10 powers of 4. This means that the rightmost two digits of powers of 4 repeat in a cycle of length 10.
Since 120 is a multiple of 10 and 127 is 7 more than a multiple of 10, the rightmost two digits

of 4^{127} are the same as the rightmost two digits of 4^7, which are 84.

The first few powers of 7 are

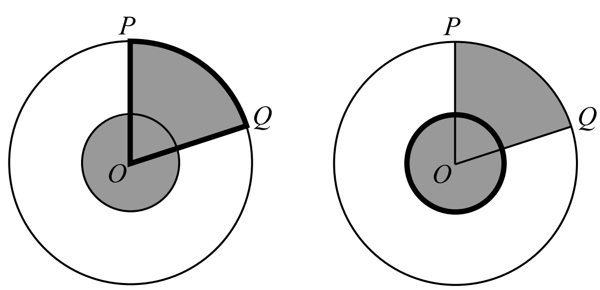
$$7^1 = 7 \qquad 7^2 = \mathbf{49} \qquad 7^3 = 3\mathbf{43} \qquad 7^4 = 24\mathbf{01} \qquad 7^5 = 16\,8\mathbf{07} \qquad 7^6 = 117\,6\mathbf{49}$$

We note that the rightmost two digits repeat after 4 powers of 7. This means that the rightmost two digits of powers of 7 repeat in a cycle of length 4.
Since 128 is a multiple of 4 and 131 is 3 more than a multiple of 4, the rightmost two digits of 7^{131} are the same as the rightmost two digits of 7^3, which are 43.

Therefore, the rightmost two digits of $4^{127} + 5^{129} + 7^{131}$ are the rightmost two digits of the sum $84 + 25 + 43 = 152$, or 52. (This is because when we add integers with more than two digits, any digits to the left of the rightmost two digits do not affect the rightmost two digits of the sum.)

ANSWER: 52

23. Since the shaded regions are equal in area, then when the unshaded sector in the small circle is shaded, the area of the now fully shaded sector of the larger circle must be equal to the area of the smaller circle.



The smaller circle has radius 1 and so it has area $\pi \times 1^2 = \pi$.
The larger circle has radius 3 and so it has area $\pi \times 3^2 = 9\pi$.
This means that the area of the shaded sector in the larger circle is π, which means that it must be $\frac{1}{9}$ of the larger circle.
This means that $\angle POQ$ must be $\frac{1}{9}$ of a complete circular angle, and so $\angle POQ = \frac{1}{9} \times 360^\circ = 40^\circ$.
Thus, $x = 40$.

ANSWER: 40

24. Since a Pretti number has 7 digits, it is of the form $a\,bcd\,efg$.
From the given information, the integer with digits abc is a perfect square.
Since a Pretti number is a seven-digit positive integer, then $a > 0$, which means that abc is between 100 and 999, inclusive.
Since $9^2 = 81$ (which has two digits) $10^2 = 100$ (which has three digits) and $31^2 = 961$ (which has three digits) and $32^2 = 1024$ (which has four digits), then abc (which has three digits) must be one of $10^2, 11^2, \ldots, 30^2, 31^2$, since 32^2 has 4 digits..
From the given information, the integer with digits $defg$ is a perfect cube.
Since the thousands digit of a Pretti number is not 0, then $d > 0$.
Since $9^3 = 729$ and $10^3 = 1000$ and $21^3 = 9261$ and $22^3 = 10\,648$, then $defg$ (which has four digits) must be one of $10^3, 11^3, \ldots, 20^3, 21^3$, since 22^3 has 5 digits.
Since the ten thousands digit and units digit of the original number are equal, then $c = g$.
In other words, the units digits of abc and $defg$ are equal.

The units digit of a perfect square depends only on the units digit of the integer being squared, since in the process of multiplication no digit to the left of this digit affects the resulting units digit.
The squares 0^2 through 9^2 are $0, 1, 4, 9, 16, 25, 36, 49, 64, 81$.
This gives the following table:

Units digit of n^2	Possible units digits of n
0	0
1	1, 9
4	2, 8
5	5
6	4, 6
9	3, 7

Similarly, the units digit of a perfect cube depends only on the units digit of the integer being cubed.
The cubes 0^3 through 9^3 are $0, 1, 8, 27, 64, 125, 216, 343, 512, 729$.
This gives the following table:

Units digit of m^3	Possible units digits of m
0	0
1	1
2	8
3	7
4	4
5	5
6	6
7	3
8	2
9	9

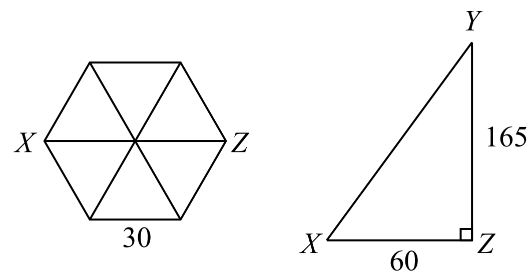
We combine this information to list the possible values of $c = g$ (from the first table, these must be 0, 1, 4, 5, 6, 9), the squares between 10^2 and 31^2, inclusive, with this units digit, and the cubes between 10^3 and 21^3 with this units digit:

Digit $c = g$	Possible squares	Possible cubes	Pretti numbers
0	10^2, 20^2, 30^2	10^3, 20^3	$3 \times 2 = 6$
1	11^2, 19^2, 21^2, 29^2, 31^2	11^3, 21^3	$5 \times 2 = 10$
4	12^2, 18^2, 22^2, 28^2	14^3	$4 \times 1 = 4$
5	15^2, 25^2	15^3	$2 \times 1 = 2$
6	14^2, 16^2, 24^2, 26^2	16^3	$4 \times 1 = 4$
9	13^2, 17^2, 23^2, 27^2	19^3	$4 \times 1 = 4$

For each square in the second column, each cube in the third column of the same row is possible. (For example, 19^2 and 11^3 give the Pretti number 3 611 331 while 19^2 and 21^3 give the Pretti number 3 619 261.) In each case, the number of Pretti numbers is thus the product of the number of possible squares and the number of possible cubes.
Therefore, the number of Pretti numbers is $6 + 10 + 4 + 2 + 4 + 4 = 30$.

ANSWER: 30

25. Throughout this solution, we remove the units (cm) as each length is in these same units.
First, we calculate the distance flown by the fly, which we call f.
Let Z be the point on the base on the prism directly underneath Y.
Since the hexagonal base has side length 30, then $XZ = 60$.
This is because a hexagon is divided into 6 equilateral triangles by its diagonals, and so the length of the diagonal is twice the side length of one of these triangles, which is twice the side length of the hexagon.

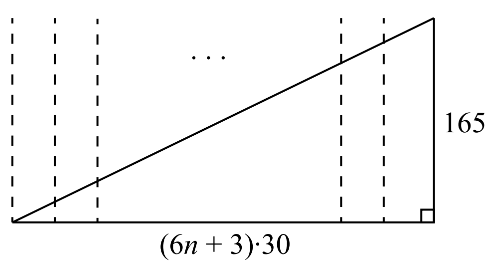


Also, $\triangle XZY$ is right-angled at Z, since XZ lies in the horizontal base and YZ is vertical.
By the Pythagorean Theorem, since $XY > 0$, then

$$XY = \sqrt{XZ^2 + YZ^2} = \sqrt{60^2 + 165^2}$$

Therefore, $f = XY = \sqrt{60^2 + 165^2}$.

Next, we calculate the distance crawled by the ant, which we call a.
Since the ant crawls $n + \frac{1}{2}$ around the prism and its crawls along all 6 of the vertical faces each time around the prism, then it crawls along a total of $6(n + \frac{1}{2}) = 6n + 3$ faces.
To find a, we "unwrap" the exterior of the prism.
Since the ant passes through $6n + 3$ faces, it travels a "horizontal" distance of $(6n + 3) \cdot 30$.
Since the ant moves from the bottom of the prism to the top of the prism, it passes through a vertical distance of 165.
Since the ant's path has a constant slope, its path forms the hypotenuse of a right-angled triangle with base of length $(6n + 3) \cdot 30$ and height of length 165.



By the Pythagorean Theorem, since $a > 0$, then $a = \sqrt{((6n + 3) \cdot 30)^2 + 165^2}$.

Now, we want a to be at least $20f$. In other words, we want to find the smallest possible value of n for which $a > 20f$.
Since these quantities are positive, the inequality $a > 20f$ is equivalent to the inequality $a^2 > 20^2 f^2$.

The following inequalities are equivalent:

$$\begin{aligned}
a^2 &> 20^2 f^2 \\
((6n+3)\cdot 30)^2 + 165^2 &> 400(60^2 + 165^2) \\
(6n+3)^2 \cdot 30^2 + 165^2 &> 400(60^2 + 165^2) \\
(6n+3)^2 \cdot 2^2 + 11^2 &> 400(4^2 + 11^2) \qquad \text{(dividing both sides by } 15^2\text{)} \\
4(6n+3)^2 + 121 &> 400 \cdot 137 \\
4(6n+3)^2 &> 54\,679 \\
(6n+3)^2 &> \tfrac{54\,679}{4} \\
6n+3 &> \sqrt{\tfrac{54\,679}{4}} \qquad \text{(since both sides are positive)} \\
6n &> \sqrt{\tfrac{54\,679}{4}} - 3 \\
n &> \tfrac{1}{6}\left(\sqrt{\tfrac{54\,679}{4}} - 3\right) \approx 18.986
\end{aligned}$$

Therefore, the smallest positive integer n for which this is true is $n = 19$.

ANSWER: 19