Answers to Practice Set Number 2

Pascal
1) C 2) E 3) D 4) D 5) C 6) B 7) E 8) E 9) D 10) D

Cayley
1) D 2) B 3) C 4) D 5) E 6) B 7) B 8) E 9) C 10) C

Fermat
1) E 2) E 3) E 4) A 5) C 6) B 7) E 8) B 9) C 10) E

Hints, suggestions, and some solutions:

Pascal

1. Since the perimeter is 64, 32 posts are required.
2. Watch for the reversal! You are not asked to 1.8% of 540.
3. Simply add $\frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{6}$.
4. The midpoint is on the x axis and so its y-coordinate is 0. Since this number is the average of the y coordinates of the end points $y = -4$.
5. Change the given ratios to $10 : 15$ and $15 : 24$ so that "y" is represented by the same quantity!
6. The left side $= 3(3^{10}) = 3^{10+1}$.
7. Since $V = LWH$, the new volume is $(1.2)L(1.2)W(1.2)H = 1.728V$.
8. Look at the areas above and below BD separately. When determine the area of unusual shapes, sometimes it helps to look for what *isn't* there.
9. Look at the prime factorization of 1872.
10. Count the triangles outward from vertex A. Do the same from vertex B but beware of the double counting.

Cayley

1. $x^2 - y^2 = 4 - 25 = -21$.
2. The intercepts of the line are 18 and -10 so the area is 90.
3. The area of the parallelogram is the rectangle's area minus that of the 4 triangles such as AXY. The area of each of the 4 triangles is $\left(\frac{1}{2}\right)\left(\frac{1}{3}\right)\left(\frac{2}{3}\right) = \frac{1}{9}$ of the rectangles area. Therefore the parallelogram has area $\frac{5}{9}$ of the rectangle.
4. We have $(a - b)x = a^2 - b^2 = (a - b)(a + b)$. So since $a - b \neq 0$, $x = a + b$.

5. Equating the left sides (since both are 1) we find that $x = y$. Substituting this into either equation $\left(\frac{5}{12}\right) x = 1$ and $x = 2.4 = y$.

6. The area of the triangle is one-half that of the parallelogram wherever we place the 4th vertex (there are three possibilities). But triangle ABC is right-angled (check slopes). Its area is $\left(\frac{1}{2}\right)(10)(10) = 50$.

7. Since $27^{27} = (3^3)^{27} = 3^{81}$, we have $3(3^{81}) = 3^{82}$.

8. The numbers 7, 8, 9 have the divisibility properties but are too small. The next set of numbers are $7 \times 8 \times 9 = 504$ greater, and are 511, 512, 513.

9. Consider the reflection of $C(8, 3)$ in the x-axis to $D(8, -3)$. The distance $BC = BD$ since B is on the x-axis. So $AB + BC = AB + BD$. But the distance $AB + BD$ is minimal when AD is straight. So just find the x intercept of AD, the line $y = -x + 5$.

10. The required figure is the diamond shape with vertices $(10, 0), (0, 10), (-10, 0)$ and $(0, -10)$. The number of points in each quadrant (not on the axes) is $1 + 2 + 3 + \cdots 99 = 4950$. There are 401 points on the axes.

Fermat

1. $(-10)^2 = 100$.

2. If $x + y = 5k$, $y + z = 11k$ and $z + x = 12k$, adding gives $2(x + y + z) = 28k$. So $x = 14k - 11k = 3k$ etc.

3. Since $28 = 2^2 \cdot 7$ we require $A = 2 \cdot 7^2 = 98$.

4. First $n = 2$. Then $35 + 2 = 15$ so $m = -10$. Then $p = -50$.

5. Triangle ACO has sides in the ratio $1 : 1 : \sqrt{2}$.

6. Use $(x + y)^2 = x^2 + y^2 + 2xy$!

7. $BD : DC = 27 : 18 = 3 : 2$. So the areas $|\triangle ABD| : |\triangle ACD| = 3 : 2$. Therefore the area of PAB is 90.

8. Multiplying gives $a^5 b^5 = 72 \cdot 108 = 2^5 \cdot 3^5$.

9. Using N, D, Q to represent the number of nickels, dimes and quarters we have $N + D + Q = 110$ and $5N + 10D + 25Q = 1000$. Multiplying the first by 10 and subtracting we get $N = 20 + 3Q$. Substituting this back $D = 90 - 4Q$. So $Q = 0, 1, 2, \ldots, 22$.

10. There are three cases

    i) Exponent 0 and base non-zero.
    ii) Base equaling 1
    iii) Base equaling -1 and exponent even.

    Each case gives 2 solutions.