Answers to Practice Set Number 4

Pascal
1) D 2) D 3) C 4) B 5) B 6) B 7) E 8) C 9) D 10) A

Cayley
1) D 2) A 3) E 4) A 5) D 6) C 7) B 8) D 9) B 10) A

Fermat
1) C 2) B 3) C 4) B 5) D 6) C 7) E 8) D 9) D 10) E

Hints, suggestions, and some solutions:

Pascal

1. Factor the denominator and then divide the common factors.
2. Think of 80% as $\frac{4}{5}$ and 125% as $\frac{5}{4}$. $\$20\ 000 \times \frac{4}{5} \times \frac{5}{4} = \$20\ 000$
3. Solve $n + (n + 2) + (n + 4) + (n + 6) + (n + 8) = 135$
4. Let n be the number, then $0.75n = 90$, so $n = 120$, and $0.5n = 60$
5. $XY^2 + YZ^2 = XZ^2$, so $YZ = 16$. Then $YM = 8$ and $XM^2 = YM^2 + XY^2$
6. $(10 \times 85 - 58)/9 = 88$
7. The probability of rolling a specific number on a die is $\frac{1}{6}$. If a 6 is rolled first, there are 3 numbers that will give a difference less than 3 (4, 5, or 6). Therefore, the probability of satisfying the condition while rolling a 6 first is $\frac{1}{6} \times \frac{3}{6} = \frac{3}{36}$
8. The sum of the 5 integers is $5 \times 20 = 100$. Since the mode is 26, there must be at least two 26s, but since the median is 22, there must only be two 26s. Thus, there are two unknown numbers, both lower than 22.
9. Let n represent the number of nickels and q represent the number of quarters. Hence, $n + q = 80$ and $5n + 25q = 1300$ or dividing this by 5 gives $n + 5q = 260$.
10. The train enters the tunnel as the second boy is $\frac{3}{4}$ from the far end of the tunnel. Thus the train travels $\frac{4}{3}$ as far as the boy in the same time. $\frac{4}{3} \times 15km/h = 20km/h$

Cayley

1. $z - x = -[(x - y) + (y - z)] = -[(-5) + (8)] = -3$
2. $a^2 - 6ab + 9b^2 = (a - 3b)^2$. Then, $(-8 - 3(-3))^2 = 1^2 = 1$
3. What is $\frac{1}{5}$ of 10^1 or 10^2?

4. Recall that $\dfrac{1}{\left(\frac{x}{y}\right)} = \dfrac{y}{x}$. For example, $\dfrac{1}{1+\frac{5}{8}} = \dfrac{1}{\left(\frac{13}{8}\right)} = \dfrac{8}{13}$.

5. $8 = 2^3$ and $4 = 2^2$. Therefore, $2^{3(x+5)} = 2^{2(4x)}$, so $3x + 15 = 8x$

6. The sum of interior angles of a polygon is equal to $180(n-2)^\circ$, where n is the number of sides. So each angle in a regular decagon is $\frac{180(10-2)^\circ}{10} = 144^\circ$. Then consider quadrilateral $ABCD$, noting that $\angle ABC = \angle BCD = 144^\circ$, and $\angle DAB = \angle CDA$.

7. The difference in x and the difference in y coordinates between the 4^{th} vertex and one of the given vertices must be the same as the same differences between the other two vertices.

8. Group the terms into blocks of 3, starting with $1 + 3 - 5$. Evaluating these blocks will give a series starting at -1 and increasing by 6 each term.

9. To determine the first digit split the 720 possible groups into 6 (number of possible digits) groups, in decreasing order of the first digit. Find what group would contain the 500^{th} term.

10. Find the point of intersection of l_2 and $y = 3x - 3$. Then use that point and the y-intercept $(0, 12)$ to determine the equation for l_1.

Fermat

1. What is $\frac{1}{2}$ of 1.0×10^1 or 1.0×10^2?

2. $(\sqrt{a+16})^2 = (\sqrt{a}+4)^2$, $a + 16 = a + 8\sqrt{a} + 16$, $0 = 8\sqrt{a}$, $a = 0$.

3. $(x^3y^2)(x^2y^3) = x^5y^5 = 2^5 \times 3^5, xy = 2 \times 3 = 6$

4. $x^2 - 2xy - 3y^2 = (x - 3y)(x + y)$. What are the factors of 21?

5. Note that $a_k = a_{k-1} + (a_{k-1} - a_{k-2}) = a_{k-1} + 6$. Since $a_1 = 5, a_n = 6n - 1$, $a_{100} = 6(100) - 1 = 599$

6. Any point equidistant from two others must lie on the perpendicular bisector of those two points, which in this case is the line $y = x$.

7. The average digit size must be $\dfrac{51}{6} = 8.5$ which implies that at least 3 digits must be 9. Possible combinations are $9, 9, 9, 9, 9, 6$; $9, 9, 9, 9, 8, 7$; $9, 9, 9, 8, 8, 8$

8. An octahedron has 12 edges. Let x be the length of one edge. $2^2 + 2^2 = x^2$, $x = 2\sqrt{2}$

9. At the vertex of the triangle drawn by the center of the circle, the circle is tangent to two sides of the original triangle. The radius from the center of the circle to this point of tangency is perpendicular to the side of the triangle. Connecting the center of the circle to the vertex of the original triangle forms a 30-60-90 triangle.

10. When rolled at 45° to the side, the ball travels the same distance horizontally as vertically. Let x be the distance traveled horizontally, then find a value of x such that $\frac{x}{4}$ and $\frac{x}{7}$ are both integers.