Faculty of Mathematics
Waterloo, Ontario N2L 3G1

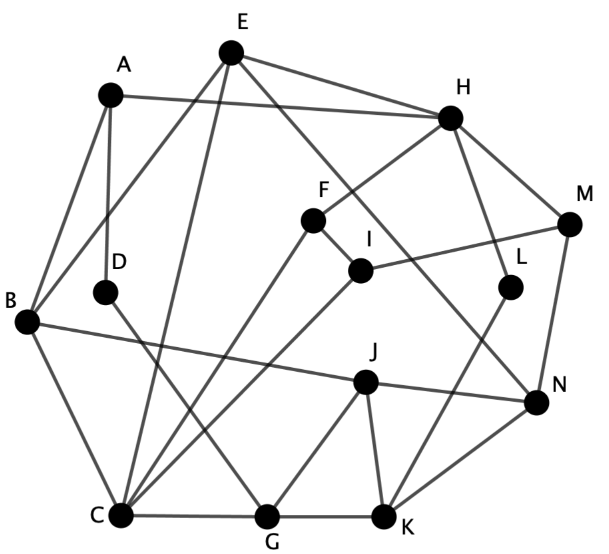
Centre for Education in
Mathematics and Computing

Grade 6 Math Circles

November 25, 2020

Graph Theory - Problem Set

1. Highlight the following walks in the graph below. For each, state whether it is a path. How do you know?



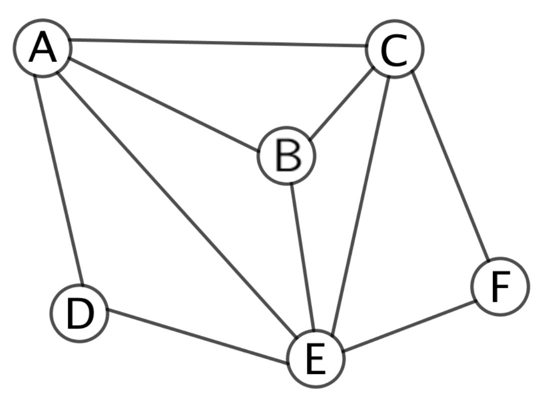
(a) A - H - L - K - J

(b) B - E - N - K - G - C -B - J

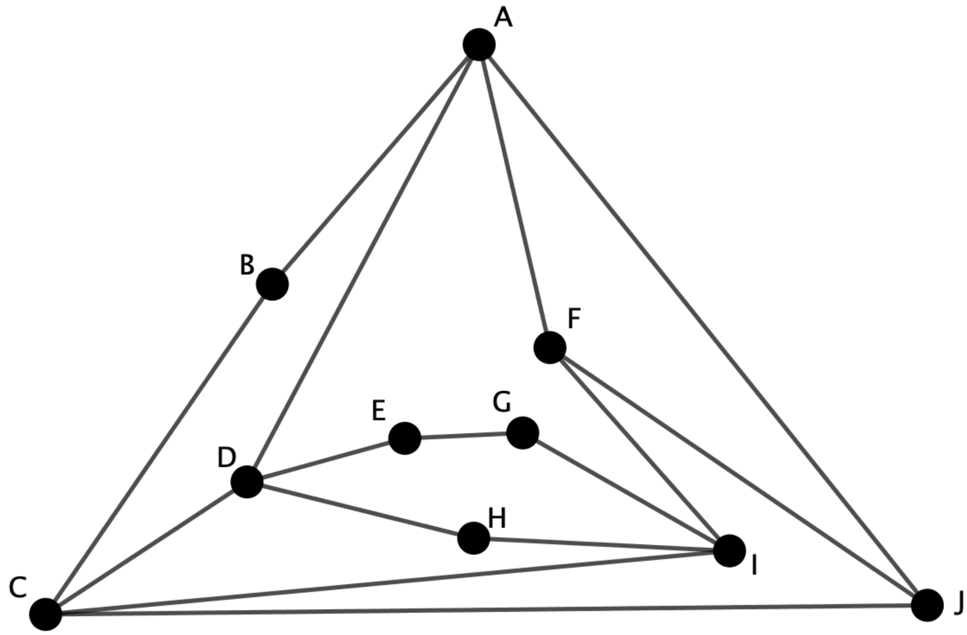
(c) M - I - F - C - E - N - J - B - A

(d) E - C - I - F - C - G - K

2. For the graph on the right:
   (a) Find all the cycles of length 3.
   Hint: There are 6 different cycles of length 3.
   (b) Find all the neighbours of vertex B.
   (c) Find a path from vertex A to vertex F.

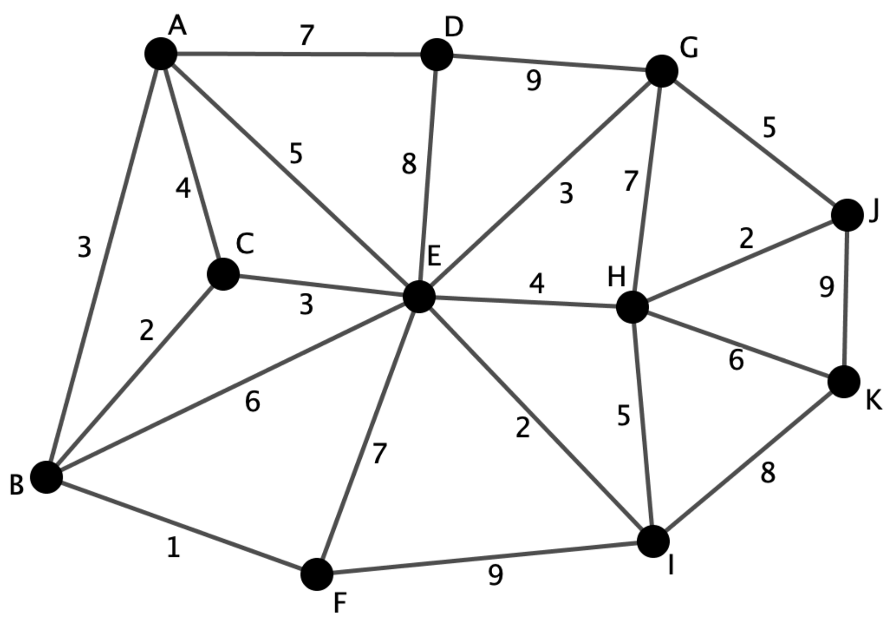


3. For the graph below, find 5 different walks from A to I. How many of those walks are paths? Find 5 paths from A to I.

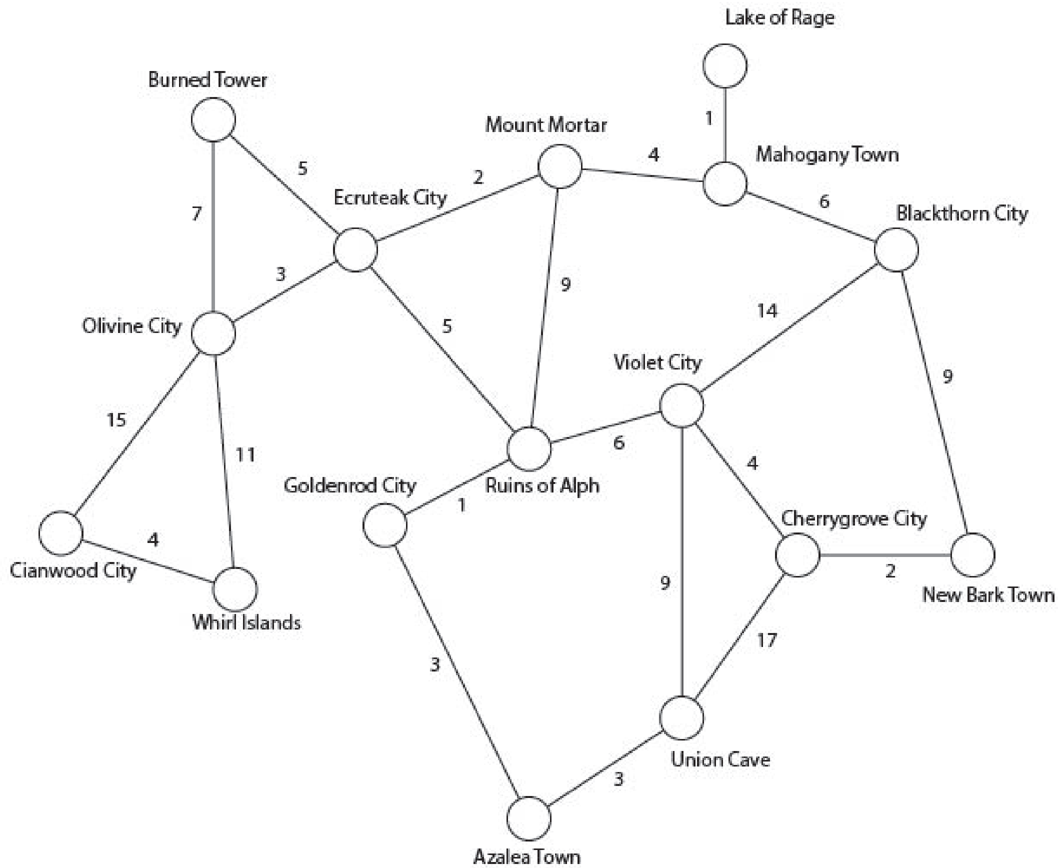


4. Use Prim's Algorithm to find the minimum spanning tree for the below graphs.
Hint: If there is a tie for the smallest edge, you can choose any of the edges with the smallest weight.

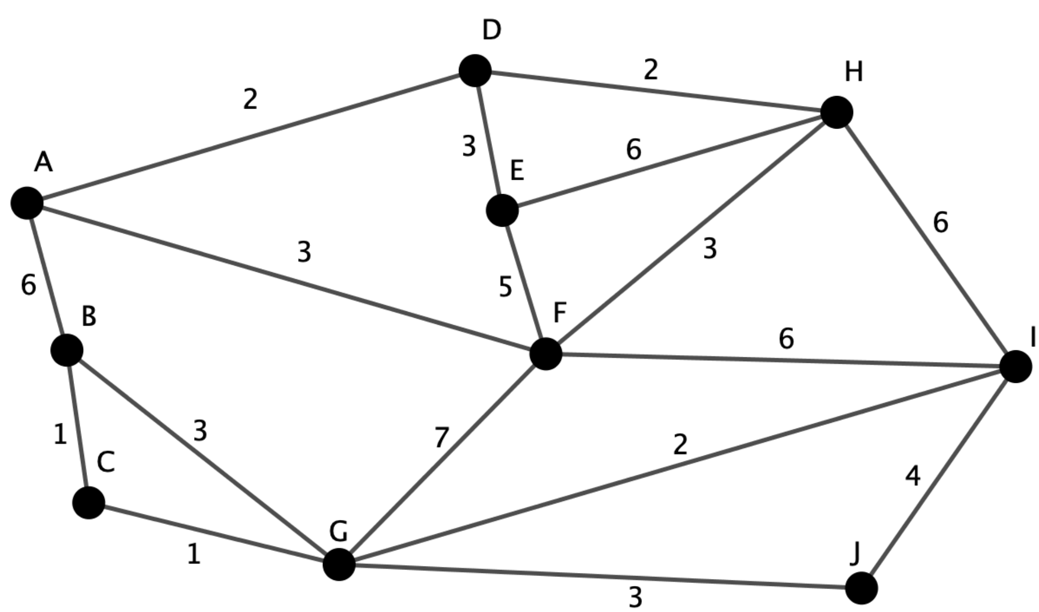
Graph 1:



Graph 2:



5. Use Dijkstra's Algorithm to find the shortest path from A to J.



6. Use Dijkstra's Algorithm twice in the below graph. Once starting at A and once starting at M. Do we get the same path from A to M as M to A? Two copies of the graph have been given. *Hint: If you get to a point where you have multiple vertices in your unvisited list with the smallest labelled cost, you can choose any of the vertices with that smallest cost to be your next current vertex.*