Grade 7/8 Math Circles
October 14, 2020
BCC Prep

Introduction

The BCC (Beaver Computing Contest) is a contest that focuses on computational and logical thinking, with no necessary coding experience. It's a way to start thinking like a Computer Scientist without having the technical background. Taking place in early November of each year, the grade 7/8 level of the contest consists of 15 questions, divided into 3 parts: A, B and C, with each getting increasingly difficult. Today, we are taking a look at some approches to solving these problems and doing some practice. More information about the contest can be found here: *https://cemc.uwaterloo.ca/contests/bcc.html*.

Warm-up

Let's start with some practice questions from previous contests. Try to solve both problems on your own and when you think you have the answer, click on the link to watch a video that briefly goes through the solutions.

Warm-up 1. *Rotation Game (BCC 2018)*

Beavers play a simple game. The game always begins with the starting position show on the right. From this starting position, rotation instructions are followed. All the rotations are clockwise and one quarter of a complete turn.
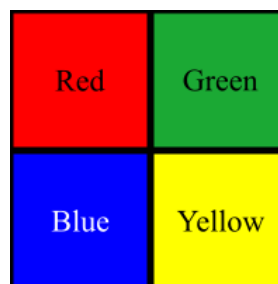
The possible instructions are:

- 1R – meaning rotate the squares one time,

- 2R – meaning rotate the squares two times,

- 3R – meaning rotate the squares three times.
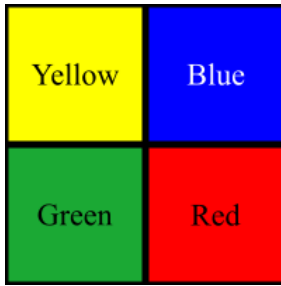
For example, if the first instruction is 2R, the top-left square will be Yellow as shown on the left. From the starting position, what colours will the top-left square be after each of the instructions 1R, 2R, 2R, and 3R are followed in order?

Warm-up 2. *Pond Planning (BCC 2016)*

Beaver neighbourhoods consist of rivers flowing between ponds. Patricia is grumpy and wants to build one dam in each neighbourhood that will cause trouble. That is, she wants to block a single river so that beavers will not be able to travel between all pairs of ponds in the neighbourhood.

In which of the following neighbourhoods is Patricia unable to build her dam?

(A)

(B)

(C)

(D)
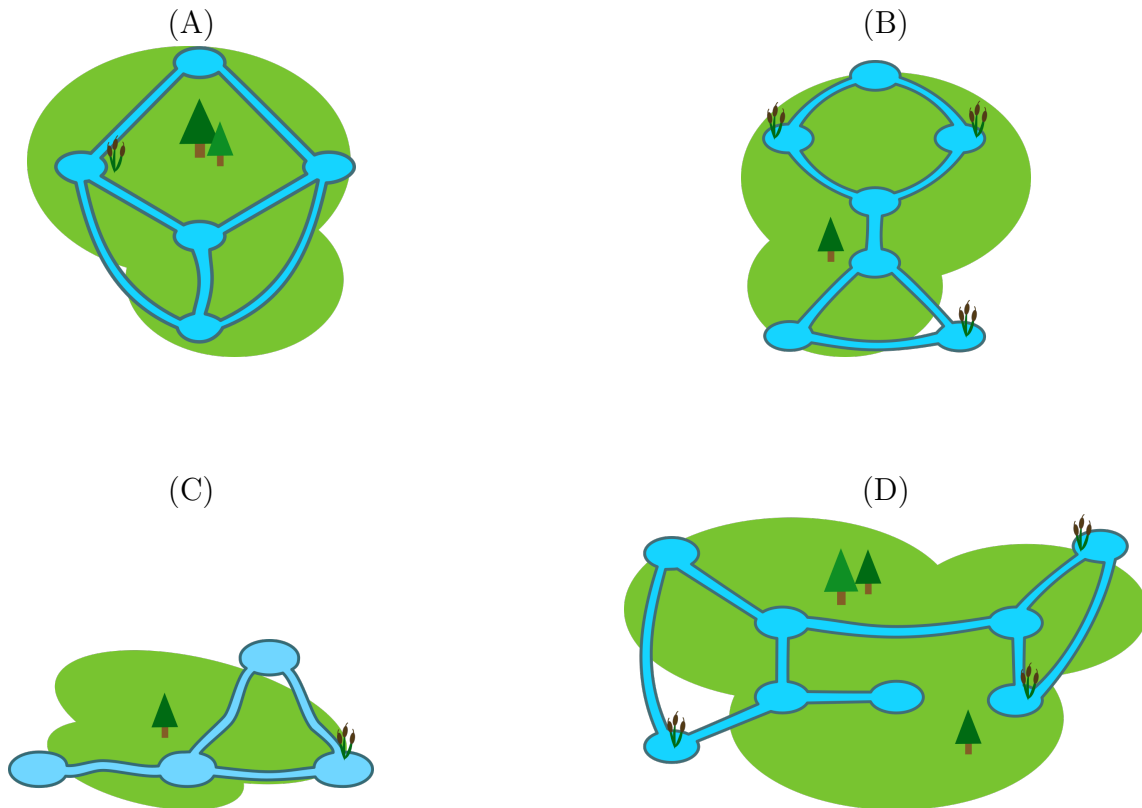
Warm-up Problem Solutions Video: *https://youtu.be/m1NLq99NLyI*

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Strategies

Although the BCC questions don't require knowledge of coding, they require logical and computational thinking. Below, a few strategies are listed to approach these types of problems. Read through the strategies, and we will put some of them into use this week.

- Each question on the BCC has a story and a question. The story will give you the background information that you need to solve the question. Start by reading the story and question, then go back and reread the story. Reread the question and story until you understand what it is asking you to find and what you are given to find it.

- After understanding the story and question, start writing down what you know. Follow that with writing down what you can deduce from what you know.

- Break the problem into pieces. Focus on one step or case at a time.

- Remove answers that are impossible and ones that you can show won't be the solution. This may be easier than solving for the solution directly.

- Make a chart or diagram to help organize what is given in the story and what you learn from it.

- Try asking the question in your own words or in a different way. You may be able to find a simpler question that, if answered, will give you the answer to the original question.

- Have fun! The BCC and other contests are all meant to be fun.

Examples

Try each of the following questions first on your own. If you get stuck, try using one of the strategies listed above. Associated with each of the following problems is a video. The video will be an explanation of one or more strategies for solving the given question. Once you have tried the problem on your own, watch the video.

Example 1. *L-Game (BCC 2016)*

Kiki and Wiwi are playing L-Game on a 4x4 board. The player who can no longer play a piece loses. They take turns placing L-shaped pieces one at a time with Kiki play first so that

- every piece placed by Kiki is oriented as shown below,

- every piece placed by Wiwi is oriented as shown below,

- every piece is placed entirely on the board, and

- no two pieces overlap.

The diagram below illustrates a possible board after each player has placed a piece once.

Kiki's orientation   First two moves   Wiwi's orientation
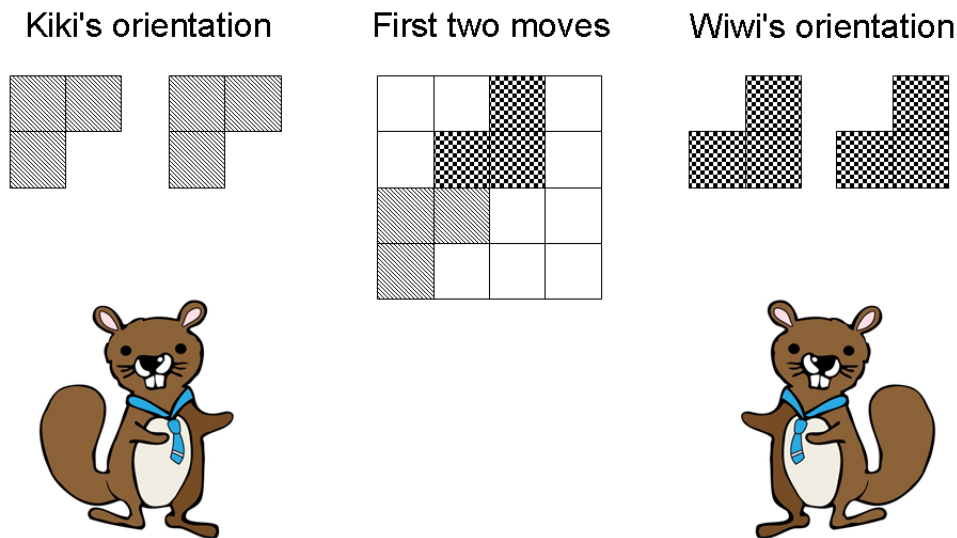
Starting from an entirely empty board, how many of Kiki's nine possible first moves guarantee that Kiki will win no matter what moves anybody makes?
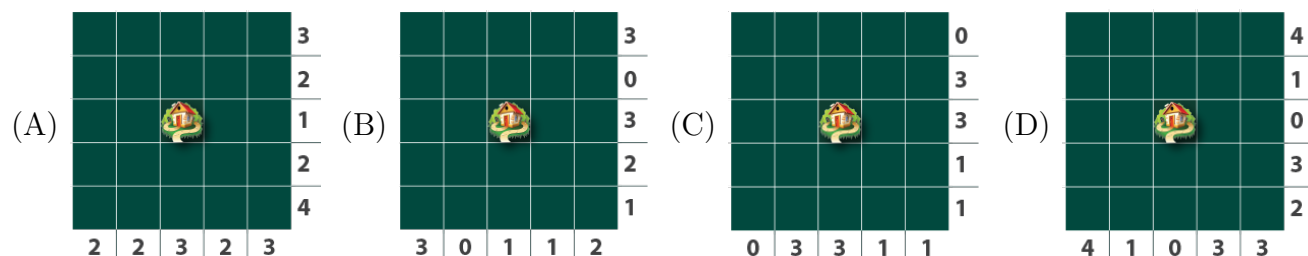
L-Game Solution Video: *https://youtu.be/R-DwEYzAgIw*

Example 2. *Farmer's Report (BCC 2019)*

Farms are divided into square fields. There is always a farmhouse in the centre square. Every year, farmers must decide whether a field will grow wheat or grass. They must report the total number of wheat fields in each row and column. An example of a report is shown on the right.
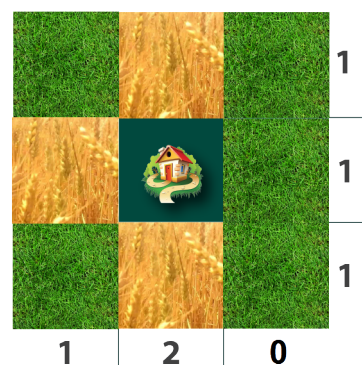
The totals given are accurate because there is one wheat field in each row, one wheat field in the left column, two wheat fields in the middle column, and no wheat fields in the right column. In each of the following reports, each dark green square, except the centre square containing the farmhouse, represents either a wheat field or a grass field. Which report could contain accurate totals?

(A) Rows: 3, 2, 1, 2, 4   Columns: 2 2 3 2 3

(B) Rows: 3, 0, 3, 2, 1   Columns: 3 0 1 1 2

(C) Rows: 0, 3, 3, 1, 1   Columns: 0 3 3 1 1

(D) Rows: 4, 1, 0, 3, 2   Columns: 4 1 0 3 3

Farmer's Report Solution Video: *https://youtu.be/bMdvK7_1-20*

Example 3. *Triple Trouble (BCC 2019)*

A beaver puts each of four toys into boxes labeled W, X, Y, and Z. Each box can hold any number of toys.
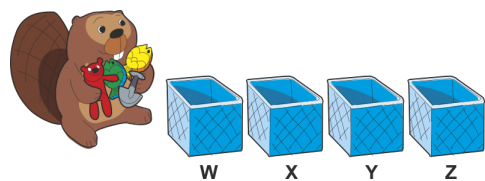
At least one of the three conditions in each row of the table shown is satisfied:

a toy is in X	no toy is in Y	no toy is in Z
a toy is in W	a toy is in X	no toy is in Z
no toy is in X	no toy is in Y	a toy is in Z
no toy is in W	no toy is in X	no toy is in Y
no toy is in X	a toy is in Y	no toy is in Z

What is the minimum possible number of empty boxes?

Triple Trouble Solution Video: *https://youtu.be/M4rhcneVqFw*