Intermediate Math Circles

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Sunken Treasure

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Sunken Treasure

Imagine that you are a deep sea diver, exploring a shipwreck for sunken treasure.

Sunken Treasure

On your first dive you find a harp, a candlestick, a teapot, some coins, and a pocket watch.

You place everything into your knapsack but the knapsack becomes too heavy to bring up to the surface.

You will have to leave some items behind, but which ones?

Sunken Treasure

Each item of treasure has a weight in grams, as well as a value in gold. Your knapsack can carry at most 2000 grams.

Can you select a subset of items such that the total value of the items chosen is maximized?

In other words, which subset of items fits in your knapsack while also being worth the most gold?

Explore this problem using the following GeoGebra app:

geogebra.org/m/hvbnqhzg

Sunken Treasure

On your second dive you find 15 items instead of 5.

Your knapsack's capacity remains the same at 2000 grams.

What is the maximum value of items you can achieve?

Which subset of items achieves this maximum value?

Explore this problem using the following GeoGebra app:

geogebra.org/m/dnrgpjcd

Problem Set

For both of the dives, answer the following questions:

1. What is the maximum value of items you can achieve?
2. Which subset of items achieves this maximum value?
3. What was your process?
4. Can you guarantee or prove that your subset is optimal? If so, how?

The Knapsack Problem

This diving scenario is an example of a classic optimization problem known as the *Knapsack Problem*.

The *Knapsack Problem* occurs under the following conditions:

1. You have a constraint.
2. You have a set of items.
3. You need to select a subset of items such that two things are simultaneously true:
    - The subset of items satisfies your constraint.
    - The value of the subset of items is the best possible value.

The Knapsack Problem

The *Knapsack Problem* has many applications including the loading of cargo onto planes, cutting fabric to minimize wastage, and investing money to maximize profit.

You may even encounter the *Knapsack Problem* on the next test you take. Given the amount of time you have left and the marks assigned to each question, which questions should you spend your time on?

Sometimes, for a specific instance, the *Knapsack Problem* can be solved fairly easily. But in general, it is a very hard problem to solve. Computer scientists and mathematicians have been studying this problem for a number of years and have not yet found an efficient algorithm that works for all instances.