CEMC Math Circles - Grade 11/12

March 3-9, 2021
Trugs and Falths

Here are the solutions for the remaining days of your travels.

Day 3: You meet Gup and Hoken.

Gup: Hoken is a Falth.
Hoken: I am a Trug or Gup is a Trug.

Solution:

Gup must be either a Trug or a Falth.

- *Suppose that Gup is a Trug.*
  This means that Gup is telling the truth and so Hoken is a Falth. Therefore, Hoken is lying and so Hoken is not a Trug and Gup is not a Trug. (Recall that if "*P* or *Q*" is false, then *P* is false and *Q* is false.) But Gup is a Trug and so we have reached a contradiction. Therefore, Gup cannot be a Trug and hence Gup must be a Falth.

- *Gup is a Falth.*
  This means that Gup is lying and so Hoken must be a Trug. Note that this information is consistent with Hoken's statement as well. Since Hoken is a Trug, Hoken must be telling the truth and indeed at least one of Gup and Hoken is a Trug. (Recall that if "*P* or *Q*" is true, then at least one of *P* and *Q* is true.)

In summary, Gup is a Falth and Hoken is a Trug.

Day 4: You meet Ized and Jeke.

Ized: Jeke is not a Falth and I am a Trug.
Jeke: Ized and I are from the same society.

Solution:

Jeke must be either a Trug or a Falth.

- *Suppose that Jeke is a Trug.*
  This means that Jeke is telling the truth. Therefore, Ized and Jeke are from the same society and so Ized is also a Trug. Note that this is consistent with what Ized said. If Ized is a Trug, then Ized is telling the truth which means Jeke is not a Falth (and so is a Trug) and Ized is also a Trug.
  Note that we did not reach a contradiction here. This means we cannot eliminate the possibility that Jeke is a Trug. Does this mean we can be sure that Jeke is a Falth? Let's confirm that the other possibility leads to a contradiction.

- *Suppose that Jeke is a Falth.*
  This means that Jeke is lying and so Jeke and Ized are from different societies. Therefore, Ized is a Trug. If Ized is a Trug, Ized is telling the truth. But this means Jeke is not a Falth. This is a contradiction. Therefore, Jeke cannot be a Falth and so must be a Trug.

Since Jeke is a Trug, we already know from our work above that Ized must also be a Trug.

In summary, Jeke and Ized are both Trugs.

Day 5: You meet Kip, Lolo and Moy.

Kip: I am not a Falth and Lolo is not a Falth.
Lolo: Kip is a Falth.
Moy: Lolo is a Trug.

Solution:

Lolo must be either a Trug or a Falth.

- *Suppose that Lolo is a Falth.*
  This means Lolo is lying and so Kip is not a Falth and is hence a Trug. If Kip is a Trug, then Kip is telling the truth and so Lolo is not a Falth. This is a contradiction. Therefore, Lolo cannot be a Falth and hence must be a Trug.

- *Lolo is a Trug.*
  This means Lolo is telling the truth and so Kip is a Falth. It also means that Moy is telling the truth about Lolo's society and hence Moy is a Trug. Note that this information is consistent with Kip's statement as well. Since Kip is a Falth, Kip must be lying and indeed the first part of Kip's statement is false.

In summary, Kip is a Falth, and Lolo and Moy are both Trugs.