



Grade 6 Math Circles
Wednesday, February 17, 2021
The Golden Ratio - Problem Set

1. What is the simplest form of the ratio 169:221?

The simplest form of 169:221 is 13:17

2. Express the following ratios as fractions in simplest form.

(a) $3:4 = \frac{3}{4}$

(c) $10:9 = \frac{10}{9}$

(e) $60:105 = \frac{4}{7}$

(b) $44:20 = \frac{11}{5}$

(d) $16:18 = \frac{8}{9}$

(f) $36:24 = \frac{3}{2}$

3. Sparky Tech's new phone charger gives 8 hours of power for every hour and a half of charging. What is the ratio of hours of charging to hours of power? (Hint: Recall that the quantities in ratios are represented using whole numbers)

To avoid fractions/decimals in our ratio, multiply 8 and $1\frac{1}{2}$ by 2. Therefore, the ratio is 3 hours of charging to 16 hours of power or 3 : 16.

4. A parking lot has four cars to every van.

- (a) Represent the number of vans to the number of cars as a ratio and as a fraction.

1:4, $\frac{1}{4}$

- (b) If there are 160 cars, how many vehicles are there in total? Find the ratio of vans to total vehicles in the parking lot.

Since $4 \times 40 = 160$, we multiply 1 by 40 to get 40 vans to 160 cars. In total, there is 200 vehicles. Then, the ratio of vans to total vehicles is 1:5 in simplest form.

- (c) What fraction of the vehicles in part (b) are vans? How does this fraction differ from the fraction you got in part (a)?

In part (b), we get that $\frac{1}{5}$ are vans. This fraction differs from the $\frac{1}{4}$ we got in part (a) because that fraction represent the comparison of the quantity of vans to cars.

5. In a vending machine, the ratio of chocolate bars to packets of gum is 2:5 and the ratio of packets of gum to energy drinks is 4:1. What is the ratio of chocolate bars to energy drinks?

The ratio of chocolate bars to packets of gum is equivalent to 8:20 and an equivalent ratio for packets of gum to energy drinks is 20:5. Since there 8 chocolate bars and 5 energy drinks for every 20 packets of gum, the ratio of chocolate bars to energy drinks is 8:5.

6. Geckos are colourful lizards with four legs and five toes on each of their feet. In one cage of geckos, you can only see their legs. You count 72 legs. How many toes do the geckos have in total?

The ratio of legs to total toes for every gecko is 4:20. Since $4 \times 18 = 72$, we multiply 20 by 18 to get that the geckos have 360 toes in total.

7. Jay's hockey team won a trophy and as a treat, her coach brought nine boxes of TimBits.

- (a) If there are three chocolate TimBits in every box of twenty TimBits, how many chocolate TimBits are there in total?

Since there are 3 chocolate TimBits in every box and there are 9 boxes so, $3 \times 9 = 27$. There are 27 chocolate TimBits in total.

- (b) What is the ratio of chocolate TimBits to total TimBits in every box? What is the ratio of total chocolate TimBits to total TimBits in all nine boxes? Simplify, if possible.

There are 3 chocolate TimBits in every box of 20 TimBits so the ratio of chocolate TimBits to total TimBits in every box is 3:20. There are 27 chocolate TimBits in total. Since there is 9 boxes of 20 TimBits, we get that there are 180 TimBits in total. So, the ratio is 27:180 which can be simplified to 3:20.

- (c) What do you notice about your answer to part (b)? The ratio of chocolate TimBits to total TimBits in every box is the same as the ratio of total chocolate TimBits to total TimBits in all 9 boxes.

- (d) Anastasia is on the team too and only likes honey curlers. She goes through all the boxes and eats only the honey curler TimBits. If she eats 18 TimBits in total, how many honey curlers were in each box?

Anastasia ate 18 honey curler TimBits from 9 boxes so, $18 \div 9 = 2$. There are 2 honey curlers in each box.

(e) Using your observation from part (c), state the ratio of total honey curlers to total TimBits in all nine boxes, in simplest form.

Since there are 2 honey curlers in every box of 20 TimBits, the ratio of total honey curlers to total TimBits in all 9 boxes is 1:10.

8. A television channel plays advertisements for 1 minute at 5-minute intervals. How many minutes of advertisement are there during a 1-hour show?

For every 5-minute interval, there is 1 minute of advertisements. So, the ratio of advertisements to total show time is 1:6. If the show is 1 hour or 60 minutes, that is 10 times the total show time in our ratio. Therefore, $1 \times 10 = 10$. There are 10 minutes of advertisements.

9. Coco's Grocery Store sells bags of chocolate chips for \$1.10. If each bag is 550 grams, what is the unit rate of grams of chocolate chips per cent?

First, convert \$1.10 into 110 cents. The current ratio of chocolate chips to cents is 550 grams : 110 cents. To find the unit rate, we need to divide both numbers by 110 which is $550 : 110 \xrightarrow{\div 110} 5 : 1$. So, the unit rate is 5 grams of chocolate chips per cent or 5 g/cent.

10. Find out the measurements of Leonardo da Vinci's most famous painting, the Mona Lisa. Is the Mona Lisa in the Golden Ratio?

The dimensions of the Mona Lisa is 77 cm by 53 cm. It is not in the Golden Ratio since the ratio between the two sides is approximately 1.45283 and the ratio of the total of both sides to the longer side is 1.68311. Note that both ratios are not the same and neither ratio is equivalent to ϕ .

11. The base of a triangle is 10 cm. If the ratio of the base to the height of the triangle satisfies the Golden Ratio, what is the height of the triangle?

Since the ratio of the base to the height of the triangle satisfies the Golden Ratio, then

$$\phi = \frac{b}{h}$$

where b is the base of the triangle and h is the height. We can solve for h since we

know that $b = 10$ cm and using ϕ .

$$\begin{aligned}\phi &= \frac{10}{h} \\ h &= \frac{10}{\phi} \\ &= 6.1803398875\dots \\ &\approx 6.18\end{aligned}$$

So, the height of the triangle is approximately 6.18 cm.

12. Start with a stick that is 50 cm long. How would you break the stick into two parts such that the ratio between the two portions is the same as the ratio between the whole stick and the larger segment i.e. the pieces are in golden ratio?

A line that is split according to the Golden Ratio has the following property: $\frac{a}{b} = \frac{a+b}{a}$ where a is the longer segment, b is the shorter segment, and $a + b$ is the length of the line. Since the fractions on both sides of the equal sign are equivalent to ϕ and we know the total length of the stick, we have

$$\begin{aligned}\phi &= \frac{a+b}{a} \\ \phi &= \frac{50}{a}\end{aligned}$$

Solving for a , we get

$$\begin{aligned}a &= \frac{50}{\phi} \\ &= 30.9016994375\dots \\ &\approx 30.9017\end{aligned}$$

If the longer segment, a , is equal to 30.9017 cm, then the shorter segment is $50 - 30.9017 = 19.083$. Therefore, the stick should be broken into two parts such that, one part is 30.9017 cm and the other is 19.083.

13. Sam creates a sequence that uses the same rule as the Fibonacci Sequence except the first two terms in his sequence are 3, 4. Find the next 5 terms of the sequence.
We are looking for the next 5 terms of the sequence given that the first two terms are 3, 4. Here are the calculations for each term:

$$\text{3rd term: } 3 + 4 = 7$$

$$\text{5th term: } 7 + 11 = 18$$

$$\text{7th term: } 18 + 29 = 47$$

$$\text{4th term: } 4 + 7 = 11$$

$$\text{6th term: } 11 + 18 = 29$$

Therefore, the next 5 terms are 7, 11, 18, 29, 47.

14. Use the first three terms of the Fibonacci Sequence to create a new sequence where the next number in the sequence is found by adding the previous three numbers. For example, $0 + 1 + 1 = 2$ so the fourth number in this sequence is 2. Calculate the next 5 terms of this sequence.

We are looking for the next 5 terms of the sequence given that the first four terms are 0, 1, 1, 2. Here are the calculations for each term:

$$\text{5th term: } 1 + 1 + 2 = 4$$

$$\text{7th term: } 2 + 4 + 7 = 13$$

$$\text{9th term: } 7 + 13 + 24 = 44$$

$$\text{6th term: } 1 + 2 + 4 = 7$$

$$\text{8th term: } 4 + 7 + 13 = 24$$

Therefore, the next 5 terms are 4, 7, 13, 24, 44.