



Grade 6 Math Circles

Wednesday, February 24, 2021

Set Theory - Solutions

Exercise 1: Write the set of days of the week using set notation. Is this a finite or infinite set?

The set of days is {Monday, Tuesday, Wednesday, Thursday, Friday, Saturday, Sunday}. This is a finite set since there are only 7 days of the week.

Exercise 2: Define a set that is not a subset of A .

An example of a set B that is not a subset of A is $B = \{0, 10\}$. Since B is not a subset of A (i.e. the elements in B are not in A), we can denote this as $B \not\subset A$.

- Describe the following sets using set notation.
 - Avengers movies - {The Avengers, Avengers: Age of Ultron, Avengers: Infinity War, Avengers: Endgame}
 - Canadian NHL teams - {Vancouver Canucks, Edmonton Oilers, Calgary Flames, Winnipeg Jets, Toronto Maple Leafs, Ottawa Senators, Montreal Canadiens}
 - 3-letter names - {Nur, Ben, Amy, Leo, Jay, Axe, Ray, Lea, Mia, ...}
 - Great Lakes - {Lake Huron, Lake Ontario, Lake Michigan, Lake Erie, Lake Superior}
 - digits of ϕ - {0, 1, 2, 3, 4, 5, 6, 7, 8, 9}
 - Fibonacci numbers - {0, 1, 2, 3, 5, 8, 13, ...}
- Let $A = \{10, 11, 12, 13, \dots, 97, 98, 99\}$. Find one way to describe the set A .

The set A can be described as the set of two-digit whole numbers.
- Give an example of each of the following:
 - a numerical finite set - the set of grade levels in Canada
 - a non-numerical finite set - the set of letters in the alphabet

- (c) a numerical infinite set - the set of multiples of 3
- (d) a non-numerical infinite set - the set of pop music artists
4. A prime number is a number that is only divisible by 1 and itself. For example, 2 is the smallest prime number because it is only divisible by 1 and 2. However, 4 is not a prime number since it is divisible by 1, 2, and 4. Let P represent the set of prime numbers. Are the following statements true or false?
- (a) $1 \in P$ False. Note that 2 is the smallest prime number and 1 is considered to be neither prime or composite (a number that is not prime).
- (b) $27 \notin P$ True. The number 27 is divisible by 1, 3, 9, 27. Thus, it is not prime and will not be an element of P .
- (c) $73 \in P$ True. The number 73 is prime.
- (d) $89 \notin P$ False. The number 89 is prime and thus, is an element of P .
- (e) P is an infinite set True. It turns out there are infinitely many prime numbers. Therefore, the set P an infinite set.
5. Let P be the set of planets in the solar system.
- (a) Describe the set using set notation. $P = \{\text{Mercury, Venus, Earth, Mars, Jupiter, Saturn, Uranus, Neptune}\}$
- (b) What is the cardinality of P ? $|P| = 8$; The cardinality of P is 8.
6. What is the smallest possible set? What is the cardinality of the set?
The smallest possible set is the empty set, the set with no elements. The cardinality of the empty set is 0 or can be written as $|\emptyset| = 0$.
7. A set has only one item in it. Are there any possible subsets? If yes, what are they?
Yes. There are two possible subsets: the empty set and the set itself.
8. Suppose a set had the following children's books by Dr. Seuss: The Lorax, Horton Hears a Who, and Fox in Socks. What is a possible larger set that this might be a subset of?
Some possible answers include: the set of Dr. Seuss books, the set of children's literature, the set of illustrated picture books, the set of poetry, the set of English books, the set of books written in the 20th century etc.

9. Given the set $A = \{1, 2, 3, 4, 5, 6, 7\}$:

(a) Is it possible to find a unique subset? In other words, does A have only one subset? Why or why not?

No. Since there are at least two possible subsets of A (e.g. the empty set and the set A itself), the set A does not have a unique subset.

(b) What is the largest possible subset of A ? What is a largest possible proper subset of A ?

The largest possible subset of A is the set A itself. So, the set $\{1, 2, 3, 4, 5, 6, 7\}$ is the largest possible subset of A . A possible largest proper subset of A is the set $\{1, 2, 3, 4, 5, 6\}$. Note that the largest proper subset of A is any subset of A with cardinality of 6.

10. Consider the following sets:

- $T = \{\text{is, and, but, it, the, my, they, weekend, hurry, noodles, jacket, a, all}\}$
- $U = \{\text{I, math, bigger, pie, nature, younger, no, have, play, legs, feel, as, that}\}$
- $V = \{\text{always, fun, better, secret, love, hill, inside, time, like, library, to, back}\}$

The set X is a proper subset of the union of T , U , and V such that, the elements in X spell out a message. What is one possible set X ? Share your answer on Piazza!

One possible set X is $X = \{\text{inside, the, library, is, a, secret, pie}\}$.

11. Let X be the set of numbers divisible by 2. Let Y be the set of numbers divisible by 3. A number is said to be divisible by 6 if it is divisible by both 2 and 3. If Z is the set of numbers divisible by 6, how can you write it using the sets X and Y ? (Hint: use set operators!)

Note that for a number to be in the set Z , it must be in both X and Y . Therefore, the set $Z = X \cap Y$.

12. A librarian put together the following sets of books:

- $A = \{\text{Harry Potter, Percy Jackson, Hunger Games, Divergent, Mortal Instruments}\}$
- $B = \{\text{Hunger Games, Divergent, The Selection, Uglies}\}$
- $C = \{\text{Harry Potter, Hunger Games, Divergent, Lord of the Rings, The Chronicles of Narnia}\}$

(a) Define the following sets:

- i. $A \cup C = \{\text{Harry Potter, Hunger Games, Percy Jackson, Divergent, Mortal Instruments, Lord of the Rings, The Chronicles of Narnia}\}$
- ii. $B \cap C = \{\text{Hunger Games, Divergent}\}$
- iii. $A \cup B = \{\text{Harry Potter, Percy Jackson, Hunger Games, Divergent, Mortal Instruments, The Selection, Ugliers}\}$
- iv. $A \cap B \cap C = \{\text{Hunger Games, Divergent}\}$

(b) Calculate the cardinality of each set. Will the cardinality of the union of the three sets, i.e. $A \cup B \cup C$, equal the sum of the cardinality of each set i.e. $|A| + |B| + |C|$? Explain.

$|A| = 5, |B| = 4, |C| = 5$; Since the three sets have common elements, the cardinality of $A \cup B \cup C$ will not equal $|A| + |B| + |C|$.

(c) Let the universal set, U , be equal to the union of all three sets. What is $(A \cup C)^c$? Note that $U = \{\text{Harry Potter, Percy Jackson, Hunger Games, Divergent, Mortal Instruments, The Selection, Ugliers, Lord of the Rings, The Chronicles of Narnia}\}$ and $A \cup C = \{\text{Harry Potter, Hunger Games, Percy Jackson, Divergent, Mortal Instruments, Lord of the Rings, The Chronicles of Narnia}\}$. The complement of $A \cup C$ will include all the elements in U that are not in $A \cup C$ itself. Therefore, we get $(A \cup C)^c = \{\text{The Selection, Ugliers}\}$.

13. If A and B are two sets such that $A \cap B = \emptyset$, what does that tell you about the sets? Recall that the symbol \emptyset represents the empty set, a set with no elements.

The two sets A and B have no elements in common.

14. The union of two sets, C and D , results in the empty set i.e. $C \cup D = \emptyset$. What are C and D ?

Since the union or combination of both sets resulted in the empty set, both sets must be empty themselves. Therefore, C and D are empty sets.

15. Sally is planning a Zoom party for her birthday. She polls her friends to find which day works best for everyone. Here are the results:

Friend	March 5	March 6	March 7	March 12	March 13	March 15
Rory	✓	✓			✓	✓
Mercury		✓	✓			✓
Lamar		✓	✓	✓	✓	
Sonia		✓	✓			✓
Amber		✓		✓		✓

- (a) Write the results as a set. Let the initial of each friend's name represent the set and list the dates that they are available e.g. the set $R = \{\text{March 5, March 6, March 13, March 15}\}$ represents the dates Rory is available.

- $M = \{\text{March 6, March 7, March 15}\}$
- $L = \{\text{March 6, March 7, March 12, March 13}\}$
- $S = \{\text{March 6, March 7, March 15}\}$
- $A = \{\text{March 6, March 12, March 15}\}$.

- (b) What set operator can be used to find which date works for everyone? Use the set operator to find the date when everyone can attend.

If we take the intersection of all the sets, we can find the date that all 5 sets have in common i.e. the date when everyone is available. The intersection of the sets is $R \cap M \cap L \cap S \cap A = \{\text{March 6}\}$. Therefore, everyone can attend on March 6.

16. Consider the universal set of integers from 0 to 100. The set E is a subset of U containing only even integers. The set O is the complement of E or $O = E^C$. Describe the set O using set notation.

Note that the set O is a subset of U containing only odd integers. So, $O = \{1, 3, 5, \dots, 93, 95, 97, 99\}$.

17. Using your knowledge of number systems, categorize the following numbers as a whole number, natural number, integer, rational number, or irrational number.

Note: use the number system that the number best falls into e.g. while -2 is technically a rational number, it is better described as an integer.

- (a) $\frac{4}{5}$ - rational number
- (b) $\sqrt{7}$ - irrational number
- (c) 3 - natural number
- (d) ϕ - irrational number
- (e) 3.14159 - rational number
- (f) -6 - integer
- (g) $-\frac{1}{3}$ - rational number
- (h) 0 - whole number