Grade 6 Math Circles
March 24, 2021
Intro to Statistics - Problem Set Solution

1. For each scenario, identify the population and sample.

   (a) A toothpaste company randomly selects 10 dentists from a province's registry of dentists to ask if they would recommend their company's toothpaste.

   > *Solution* : The population is all dentists on the province's registry, and the sample is the 10 randomly selected dentists.

   (b) A food safety inspector randomly checks the temperature of 20 meals that exit the restaurant kitchen during the hour from 6–7pm to see if they have been cooked to a safe temperature.

   > *Solution* : The population is all the meals that exited that restaurant kitchen during the hour from 6–7pm, and the sample is the 20 randomly selected meals that the inspector checked during the hour from 6–7pm.

   (c) A fast food company randomly selects 10 of their own restaurants each month. They then interview all of the employees at that location about their work environment.

   > *Solution* : The population is all of the employees at the company's restaurants, and the sample is all of the employees at the 10 randomly selected restaurants.

   (d) A hockey equipment manufacturer wants to learn about who is buying their equipment online, so with every tenth purchase, they ask the customer a few simple questions before checking out.

   > *Solution* : The population is everyone who buys the manufacturer's equipment online, and the sample is every tenth purchaser.

(e) A quality control inspector randomly selects 50 cans of soda from a factory production line each day to check for defects.

> *Solution* : The population is all cans of soda produced on the factory production line on a given day, and the sample is the 50 randomly selected soda cans that day.

2. For each scenario, identify whether it would be more appropriate to use a census or a sample survey to collect data. Why?

(a) A team of researchers wants to learn more about the relationship between exercise and pet ownership to design a program to improve people's health.

> *Solution* : A sample survey would be more appropriate. It would be infeasible to survey the entire population of "people", both because it is difficult to clearly define, and because it is a very large group.

(b) To assure the quality of their product, a factory must open individual packages for inspection.

> *Solution* : A sample survey would be more appropriate. It's not specified how many packages are produced at the factory, so it's not obvious whether the population is large or not. However, it is not feasible to open every single package produced—then no products could actually be sold!

(c) You have some friends over and are collecting orders for lunch.

> *Solution* : A census would be more appropriate. A group of friends is not very large, and it makes sense here to have one response from each person in the population (you and your friends) in the context of ordering lunch.

(d) An advertising agency wants to test a new marketing campaign before launching it.

> *Solution* : A sample survey would be more appropriate. The population, which would be the entire potential audience for the marketing campaign, is difficult to clearly define and likely very large. Additionally, it does not make sense to test a marketing campaign on everybody in a population before launching it to the same population.

(e) A large conference needs to compile a list of email addresses to send out informa-
tion in advance.

> *Solution* : A census would be more appropriate. Regardless of how large
> the population is, it makes sense to gather a response from every individual.
> Additionally, it is not challenging to gather these responses, as attendees
> presumbly need to register for the conference somehow if they wish to at-
> tend.

3. Identify each variable as quantitative or categorical.

(a) the height of a tree

> *Solution* : quantitative.

(b) the variety (type) of an apple

> *Solution* : categorical.

(c) the breed of a dog

> *Solution* : categorical.

(d) the city that a person was born in

> *Solution* : categorical.

(e) the amount of water that a cup holds

> *Solution* : quantitative.

(f) the price of a hotdog

> *Solution* : quantitative.

(g) the favourite number of a person

> *Solution* : categorical or quantitative, depending on what the context of
> the survey is!

(h) the age of a person, grouped into a class: 0–9, 10–19, 20–29, and so on

> *Solution* : categorical.

(i) the length of a movie

> *Solution* : quantitative.

(j) the credit card number of a person

> *Solution* : categorical.

4. Find the mean of each list of numbers.

   (a) 9, 5, 10, 6, 7

   > *Solution* : $\frac{9+5+10+6+7}{5} = \frac{37}{5} = 7.4$

   (b) 11, 16, 11, 12, 18, 9, 12, 14, 15, 12

   > *Solution* : $\frac{11+16+11+12+18+9+12+14+15+12}{10} = \frac{130}{10} = 13$

   (c) 87, 69, 70, 72, 94, 85

   > *Solution* : $\frac{87+69+70+72+94+85}{6} = \frac{477}{6} = 79.5$

5. A school has 400 students in total. Find the proportion of students in each grade. Express this proportion as a simplified fraction, a decimal, and a percentage.

grade	# of students in grade	proportion of students in grade
5	93	?
6	125	?
7	88	?
8	94	?

> *Solution* :
>
> | grade | # of students in grade | fraction | decimal | percentage |
> |---|---|---|---|---|
> | 5 | 93 | $\frac{93}{400}$ | 0.2325 | 23.25% |
> | 6 | 125 | $\frac{125}{400} = \frac{5}{16}$ | 0.3125 | 31.25% |
> | 7 | 88 | $\frac{88}{400} = \frac{11}{50}$ | 0.22 | 22% |
> | 8 | 94 | $\frac{94}{400} = \frac{47}{200}$ | 0.235 | 23.5% |

6. Your class is responsible for planning a surprise ice cream party at school. There are four flavours of ice cream to choose from: vanilla, chocolate, strawberry, and mint chocolate-chip. Each tub of ice cream will be be enough for 30 sundaes. Altogether, there are 800 students at your school to order ice cream for. It's okay to have a bit extra, but we want to end up with as little waste as we can.

To decide how many tubs of each flavour to order, you conduct a survey in your class of 20 students. Out of the 20 students, 4 choose vanilla, 6 choose chocolate, 3 choose strawberry, and 7 choose mint chocolate-chip.

(a) What are the population and sample?

> *Solution* : The population is the 800 students at your school. The sample is the 20 students in your class who you survey.

(b) Is the variable quantitative or categorical?

> *Solution* : The variable, ice cream flavour, is categorical. There are four categories: vanilla, chocolate, strawberry, and mint chocolate-chip.

(c) Based on your sample survey, how many tubs of each flavour of ice cream should you order?

> *Solution* : First, we'll find the answer for vanilla ice cream.
>
> First, we calculate the proportion of surveyed students who chose each flavour. Then, we calculate how many students each proportion represents in the population by multiplying the proportion and the size of the population.
> $\frac{4}{20} \times 800 = 160$
>
> Next, we calculate how many tubs of ice cream we would need to make a sundae for each of the students who prefer that flavour.
> $160 \div 30 = 5\frac{1}{3}$
>
> This means that we should round up and order 6 tubs of vanilla ice cream, since we can only order whole tubs. Now, we repeat these steps with the other flavours.
>
> *(continued on next page)*

Chocolate: $\frac{6}{20} \times 800 = 240$

$240 \div 30 = 8$ tubs of chocolate ice cream.

Strawberry: $\frac{3}{20} \times 800 = 120$

$120 \div 30 = 4$ tubs of strawberry ice cream.

Mint chocolate-chip: $\frac{7}{20} \times 800 = 280$

$280 \div 30 = 9\frac{1}{3}$ which rounds to 10 tubs of mint chocolate-chip ice cream.

(d) After every students gets one sundae, how many will be left over?

Solution : Altogether, we would order $6+8+4+10 = 28$ tubs of ice cream, which is enough for $28 \times 30 = 840$ sundaes. Thus, $840 - 800 = 40$ sundaes would be left over.

7. A small start-up company, Alarm, wants to survey its customers about satisfaction with their product. Altogether, Alarm has 90 000 customers. A survey is sent to 4500 of them. However, in the end, only 1000 of those customers filled it out.

The responses are summarised below:

rating	# of people
5	349
4	428
3	74
2	24
1	125

(a) What are the population and sample?

Solution : The population is all Alarm customers. The sample is the 1000 customers who filled out the survey. (The sample intended by the surveyers was the 4500 customers contacted, but 3500 of those customers did not respond, and thus no data was collected from them.)

(b) Is the variable quantitative or categorical?

> *Solution* : The variable is both quantitative and categorical! It is a number from 1 to 5 which makes sense to do math with. However, we can also treat each of the five possible ratings as their own categories.

(c) Based on the sample data, what is the average customer satisfaction of Alarm products?

> *Solution* : $((5 \times 349) + (4 \times 428) + (3 \times 74) + (2 \times 24) + (1 \times 125)) \div 1000$
> $= (1745 + 1712 + 222 + 48 + 125) \div 1000$
> $= 3852 \div 1000$
> $= 3.852$

(d) What proportion of customers rate Alarm products as 3 out of 5 or lower?

> *Solution* : $(74 + 24 + 125) \div 1000 = \frac{223}{1000} = 0.223 = 22.3\%$

8. To compete in a school Halloween candy contest, students have to guess how many of each type of candy bar are in a bucket. There are three types of candy bars—Jupiter Bars, Saturn Bars, and Neptune Bars—and 200 total candies in the bucket. Before guessing, each student is allowed to close their eyes and randomly pick 8 candies out of the bucket to inform their guess. The student who guesses the closest for how many of each bar there are in the bucket gets to take all of the candy home.

During your turn, you get 3 Jupiter Bars, 3 Saturn Bars, and 2 Neptune Bars.

(a) What are the population and sample?

> *Solution* : The population is all of the candy bars in the bucket. The sample is the 8 bars you get to randomly pick out of the bucket.

(b) Is the variable quantitative or categorical?

> *Solution* : The variable, which is the kind of candy bar, is categorical.

(c) Based on your sample, what should you guess for how many of each type of candy bar there is?

> *Solution* : We approach this by calculating the proportion of each type of candy bar in the sample, and then calculating how many candy bars that proportion represents in the population.
>
> Jupiter Bars: $\frac{3}{8} \times 200 = 75$
>
> Saturn Bars: $\frac{3}{8} \times 200 = 75$
>
> Neptune Bars: $\frac{2}{8} \times 200 = 50$
>
> Thus, your best guess based on the sample would be 75 Jupiter Bars, 75 Saturn Bars, and 50 Neptune Bars.

9. For an activity in class, every student randomly picks out 10 books from your school library and calculates the average number of pages in their books. The book lengths of you and two classmates are summarised below:

| | | # of pages: | |
book	you	classmate A	classmate B
1	23	48	48
2	72	48	85
3	108	18	218
4	98	320	30
5	16	510	373
6	210	280	188
7	208	52	250
8	755	366	222
9	108	324	384
10	76	288	288

(a) What are the population and sample for you, Classmate A, and Classmate B?

> *Solution* : The population is all of the books in the library. The sample for you is the 10 books that you picked. The sample for Classmate A is the 10 books that Classmate A picked. The sample for Classmate B is the 10 books that Classmate B picked.

(b) Is the variable quantitative or categorical?

> *Solution* : The variable, which is the number of pages in the book (the length of the book), is quantitative.

(c) What is the average length of the 10 books that you picked out?

Solution : $(23 + 72 + 108 + 98 + 16 + 210 + 208 + 755 + 108 + 76) \div 10$

$= 1674 \div 10$

$= 167.4$ pages.

(d) What is the average length of the 10 books that Classmate A picked out?

Solution : $(48 + 48 + 18 + 320 + 510 + 280 + 52 + 366 + 324 + 288) \div 10$

$= 2254 \div 10$

$= 225.4$ pages.

(e) What is the average length of the 10 books that Classmate B picked out?

Solution : $(48 + 85 + 218 + 30 + 373 + 188 + 250 + 222 + 384 + 288) \div 10$

$= 2086 \div 10$

$= 208.6$ pages.

(f) Compare the three averages. Why are they the same or different? If they are different, are they close? Why?

Solution : The three averages are different, but somewhat close to each other (167.4 pages, 225.4 pages, and 208.6 pages). Even though the population is the same for everybody, it makes sense that students would get different averages, because the sample they each chose is probably going to have different books in it. However, the three averages are still somewhat close to each other, since they all picked books from the same library, and the shorter and longer books balance each other out in each sample.

(g) Try the same activity! Close your eyes and point to a book to add it to your sample of 10 books, and then find the average length of books in your sample. Chose another sample of 10 books and compare the averages lengths in your two samples.

Solutions will vary.

10. Look up the latest reports of COVID-19 cases in your area. It'll also be helpful to find out your area's population!

   (a) Based on the the information available about the number of total cases, what is the proportion of people in your area who have tested positive for COVID-19

   > *Solutions will vary.*
   > *Example solution:* Winnipeg has a population of 749 534 (as of 2017). There have been 32 903 reported cases since records began, which gives us a proportion of $\frac{32,903}{749,534}$, which is approximately $0.044 = 4.4\%$ of people in the population.

   (b) Based on the the test positivity rate (which is the proportion of tests taken that had positive results for the virus) and total cases, how many COVID-19 tests have been administered in your area?

   > *Solutions will vary.*
   > *Example solution:* Winnipeg has a population of 749 534 (as of 2017). The current test positivity rate in Manitoba (the province that Winnipeg is in) is 5.2%. Thus, around $32,903 \div 0.052 \approx 632,788$ COVID-19 tests have been administered in Winnipeg.