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Grade 7/8 Math Circles

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Squares & Radicals - Solutions

1. Evaluate.

   (a) $5^3 = 125$

   (b) $\left(\frac{101}{4}\right)^0 = 1$

   (c) $2^6 = 64$

   (d) $1^{27} = 1$

   (e) $4^4 = 256$

   (f) $0^{56} = 0$

   (g) $13^2 = 169$

   (h) $\phi^1 = \phi$

2. Write the following as powers. *(Hint: For part (c), try factoring the number)*

   (a) $333 \times 333 \times 333 = 333^3$

   (b) 91 to exponent 5 $= 91^5$

   (c) $216 = 6^3$

   (d) 5 to the fourth power $= 5^4$

   (e) 100 squared $= 100^2$

   (f) $7 \times 11 \times 11 \times 7 \times 11 \times 7 \times 7 = 7^4 \times 11^3$

3. Use the given base to write the following numbers as powers.

   (a) 4096, base $= 4$; $4096 = 4^6$

   (b) 625, base $= 5$; $625 = 5^4$

   (c) 1000, base $= 10$; $1000 = 10^3$

   (d) 512, base $= 2$; $512 = 2^9$

   (e) 289, base $= 17$; $289 = 17^2$

   (f) 81, base $= 3$; $81 = 3^4$

4. Express 1 as a power that has a base not equal to 1.
   Recall that any base, not including 0, raised to the exponent 0 has value 1. So, one possible answer is 123^0.

5. Evaluate.

   (a) $(-30)^4 = 30^4 = 810,000$

   (b) $47^{-1} = \left(\frac{1}{47}\right)^1 = \frac{1}{47}$

   (c) $2^{-6} = \left(\frac{1}{2}\right)^6 = \frac{1^6}{2^6} = \frac{1}{64}$

   (d) $(-1)^{38} = 1^{38} = 1$

(e) $(15)^{-3} = \left(\frac{1}{15}\right)^3 = \frac{1^3}{15^3} = \frac{1}{3375}$

(f) $\left(-\frac{7}{6}\right)^5 = \left(-\frac{7^5}{6^5}\right) = \left(-\frac{16,807}{7,776}\right)$

(g) $(-88)^0 = 1$

(h) $\left(-\frac{4}{25}\right)^{-2} = \left(-\frac{25}{4}\right)^2 = \left(\frac{25}{4}\right)^2 = \frac{25^2}{4^2} = \frac{625}{16}$

6. Fill in the table summarizing different properties of exponents.

Property	Explanation	Example
Base of 1	A power with the base 1 will equal 1 for any exponent.	$1^{23456789} = 1$
Base of 0	A power with the base 0 will equal 0 for all exponents except for 0.	$0^{1000} = 0$
Exponent of 1	Any base to the exponent 1 will simply return the value of the base itself.	$\pi^1 = \pi$
Exponent of 0	Any base that is not equal to 0 will equal 1 when raised to the exponent 0.	$\left(-\frac{359^2+e}{88}\right)^0 = 1$

Negative Exponent	When a power has a negative exponent, we raise the reciprocal of the base to the corresponding positive exponent.	$10^{-5} = \left(\frac{1}{10}\right)^5 = \frac{1}{100,000}$ $\left(\frac{14}{11}\right)^{-2} = \left(\frac{11}{14}\right)^2 = \frac{121}{196}$
Negative Base	A negative base to an odd exponent will result in a negative answer while a negative base to an even exponent will result in a positive answer.	$(-7)^3 = (-1) \times (7)^3 = (-1) \times 343 = -343$ $\left(-\frac{3}{4}\right)^6 = \left(\frac{3}{4}\right)^6 = \frac{729}{4096}$

7. Consider the fraction $\frac{16}{81}$.

   (a) Write $\frac{16}{81}$ as a power with a negative base.

   Note that $\frac{16}{81}$ is $\left(\frac{2}{3}\right)^4$. Since 4 is an even exponent, the power $\left(-\frac{2}{3}\right)^4$ can also be simplified to $\frac{16}{81}$. Therefore, $\frac{16}{81}$ can be written as the power $\left(-\frac{2}{3}\right)^4$

   (b) Write the fraction as a power with a negative base and exponent.

   From part (a), we know that the fraction can be written as the power $\left(-\frac{2}{3}\right)^4$. If we make the exponent negative, we need to take the reciprocal of the base to retain the same value. So, the fraction can be written $\left(-\frac{3}{2}\right)^{-4}$

8. Fill in the blanks with $<$, $>$, or $=$ to complete the inequality.

(a) $2^4 = 4^2$

(b) $6^{-3} < 6^3$

(c) $99^0 > 0^{45}$

(d) $(-1)^{27} < 1^{27}$

(e) $7^{-3} < 7^{-1}$

(f) $4^{-6} > (-4)^{-7}$

9. The following table lists the first 20 perfect squares. Fill in the blanks. The table can be completed online at https://www.geogebra.org/m/pkcuyncv.

x	1	2	3	4	5	6	7	8	9	10
x^2	1	4	9	16	25	36	49	64	81	100
x	11	12	13	14	15	16	17	18	19	20
x^2	121	144	169	196	225	256	289	324	361	400

10. Rubik's Cubes are popular puzzle toys made up of 3 by 3 miniature cubes or "cubelets". A cubelet on a Rubik's Cube has side length 1.9 cm.

(a) What is the area of one square surface on the cubelet?

The formula for area of a square is $A = s^2$ where A is the area and s denotes the side length of the square. Since the side length is 1.9 cm, then $A = (1.9)^2$. Therefore, the area of one square surface is 3.61 cm^2.

(b) What is the surface area of the cubelet?

The formula for surface area, SA, of a cube is $SA = 6s^2$ or 6 times the area of one square face on the cube. In part (a), we calculated the area of one square surface to be 3.61 cm^2. So, the surface area of the cubelet is 6 times 3.61 or 21.66 cm^2.

(c) What is the surface area of the Rubik's Cube?

To calculate the surface area of the Rubick's Cube, first calculate the area of one face of the cube. Then, multiply it by the 6 faces of the cube (similar to part (b)). Since the Rubik's Cube is made up of 3 by 3 cubelets, one face of the Rubik's cube is made up of 9 square cubelet surfaces. So, we can find the area by multiplying the area of one square surface of the cubelet (from part (a)) by 9. Therefore, one face of the Rubik's Cube is 3.61×9 or 32.49 cm^2. Now, multiplying this area by 6, we get that the surface area of the Rubik's Cube is 292.41 cm^2.

11. A cube box has volume 125 cm^3. What are the dimensions of the box?

To compute the volume, V, of a cube, we use the formula $V = s^3$ where s is the side length of the cube. We are given the volume of the cube box to be 125 cm^3. In other words, $125 = s^3$. If we take the cube root of 125 or $\sqrt[3]{125}$, we get that the side length of the cube is 5. Thus, the dimensions of the box are 5 cm by 5 cm by 5 cm.

12. Sunil claims that powers can be used to calculate the perimeter of squares in addition to the area. He says that the formula for the perimeter of a square can be expressed

as $P = s^4$ where s is the side length of the square. Is he correct? How do you know?
Sunil is incorrect in his claim. The formula for the perimeter, P, of a square with side length, s, is $P = s + s + s + s$. To denote repeated addition, we use multiplication not exponentiation. So, we can rewrite the perimeter as $P = 4s$ not $P = s^4$.

13. Evaluate.

(a) $\sqrt[3]{4096} = 16$

(b) $\sqrt[8]{0} = 0$

(c) $\sqrt{625} = 25$

(d) $\sqrt[4]{1} = 1$

(e) $\sqrt[3]{3375} = 15$

(f) $\sqrt[6]{\frac{1}{64}} = \frac{1}{2}$

(g) $\sqrt[9]{512} = 2$

(h) $\sqrt{\frac{49}{256}} = \frac{7}{16}$

14. Simplify.

(a) $\sqrt{529} = 23$

(b) $\sqrt{675} = \sqrt{225} \times \sqrt{3} = 15\sqrt{3}$

(c) $\sqrt{37} = \sqrt{37}$

(d) $\sqrt{156} = \sqrt{4} \times \sqrt{39} = 2\sqrt{39}$

(e) $\sqrt{224} = \sqrt{16} \times \sqrt{14} = 4\sqrt{14}$

(f) $\sqrt{\frac{5}{36}} = \frac{\sqrt{5}}{\sqrt{36}} = \frac{\sqrt{5}}{6}$

(g) $13\sqrt{168} = 13(\sqrt{4} \times \sqrt{42} = 13 \times 2\sqrt{42} = 26\sqrt{42}$

(h) $\frac{1}{2}\sqrt{\frac{4}{25}} = \frac{1}{2}\left(\frac{\sqrt{4}}{\sqrt{36}}\right) = \frac{1}{2} \times \frac{2}{5} = \frac{1}{5}$

15. Is 2400 is a perfect square? Explain. *(Hint: Simplify the radical)*
If 2400 is a perfect square, then $\sqrt{2400}$ can be simplified to a whole number. So,

$$\begin{aligned}\sqrt{2400} &= \sqrt{400} \times \sqrt{6} \\ &= 20 \times \sqrt{6} \\ &= 20\sqrt{6}\end{aligned}$$

Since $\sqrt{2400}$ cannot be simplified further than $20\sqrt{6}$, we can conclude that 2400 is not a perfect square.

16. The Titans football field is $98\sqrt{3}$ m by $42\sqrt{5}$ m. What is the area of the football field?
Note that the football field is a rectangle. So, we use the formula $A = l \times w$ where A

is the area, l is the length, and w is the width. Then,

$$\begin{aligned} A &= l \times w \\ &= 98\sqrt{3} \times 42\sqrt{5} \\ &= (98 \times 42)(\sqrt{3} \times \sqrt{5}) \\ &= 4116\sqrt{15} \end{aligned}$$

Therefore, the area of the football field is $4116\sqrt{15}$ m^2.

17. The surface area of a square cake is 564 cm^2.

    (a) What is the side length of the cake?
    Recall that the formula for surface area of a cube is $SA = 6s^2$ where s is the side length of the cube. In other words, the surface area of a cube can be calculated by multiplying the area of one square surface by 6. If we reverse the process and divide the total surface area by 6, we get the area of one square surface. Thus, the area of one face of the cube is $564 \div 6$ or 94 cm^2. To find the side length of the cube, we then compute the square root of 94. Observe that 94 is not a perfect square and the radical, $\sqrt{94}$ cannot be simplified since 94 has no factors that are perfect squares. Therefore, the side length of the cake is $\sqrt{94}$ cm.

    (b) Round the dimensions of the cake to the nearest whole number. What is the new surface area?
    The decimal expansion of $\sqrt{94}$ is 9.695359715.... When rounded to the nearest whole number, the side length of the cake is 10 cm and the dimensions of the cake are 10 cm by 10 cm by 10 cm. Using the surface area formula, we get that the new surface area of the cake is 600 cm^2.

    (c) Jackie wants a cake that is $9\sqrt{30}$ cm by $9\sqrt{30}$ cm by 18 cm. What is the surface area of the cake? What is the volume?
    Using the formula $SA = 2lw+2wh+2lh$ such that, l is the length, w is the width, and h is the height, we get

$$\begin{aligned} SA &= 2(9\sqrt{30})(9\sqrt{30}) + 2(9\sqrt{30})(18) + 2(9\sqrt{30})(18) \\ &= 2((9 \times 9)(\sqrt{30} \times \sqrt{30}) + 2((9 \times 18)\sqrt{30}) + 2((9 \times 18)\sqrt{30}) \\ &= 2((81)(30)) + 2(162\sqrt{30}) + 2(162\sqrt{30}) \\ &= 2(2430) + 324\sqrt{30} + 324\sqrt{30} \\ &= 4860 + 648\sqrt{30} \ \textit{Note: This cannot be simplified further.} \end{aligned}$$

Using the formula $V = lwh$ such that, l is the length, w is the width, and h is the height, we get

$$\begin{aligned}
V &= 9\sqrt{30} \times 9\sqrt{30} \times 18 \\
&= (9 \times 9)(\sqrt{30} \times \sqrt{30}) \times 18 \\
&= (81)(30) \times 18 \\
&= 2430 \times 18 \\
&= 43,740
\end{aligned}$$

Therefore, the surface area of Jackie's cake is $4860 + 648\sqrt{30}$ cm^2 and the volume is $43,740$ cm^3.