Intermediate Math Circles
FGH
Problem Set

Problem 1
By finding a common denominator, we see the $\frac{1}{3}$ is greater than $\frac{1}{7}$ because $\frac{7}{21} > \frac{3}{21}$. Similarly, we see that $\frac{1}{3}$ is less than $\frac{1}{2}$ because $\frac{2}{6} < \frac{3}{6}$.

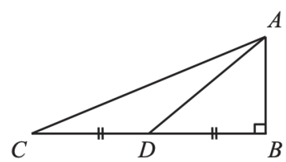
(a) Determine the integer n so that $\frac{n}{40}$ is greater than $\frac{1}{5}$ and less than $\frac{1}{4}$.

(b) Determine all possible integers m so that $\frac{m}{8}$ is greater than $\frac{1}{3}$ and $\frac{m+1}{8}$ less than $\frac{2}{3}$.

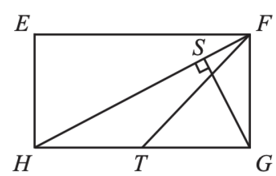
(c) Fiona calculates her *win ratio* by dividing the number of games that she won by the total number of games that she has played. At the start of a weekend, Fiona has played 30 games, has w wins, and her win ratio is greater than 0.5. During the weekend, she plays five games and wins three of these games. At the end of the weekend, Fiona's win ratio is less than 0.7. Determine all values of w.

Problem 2
A *median* is a line segment drawn from a vertex of a triangle to the midpoint of the opposite side of the triangle.

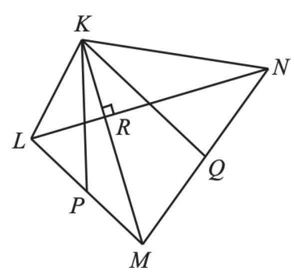
(a) In the diagram, $\triangle ABC$ is right-angled and has side lengths $AB = 4$ and $BC = 12$. If AD is the median of $\triangle ABC$, what is the area of $\triangle ACD$.



(b) In rectangle $EFGH$, point S is on FH with SG perpendicular to FH. In $\triangle FGH$, median FT is drawn as shown. If $FS = 18, SG = 24$ and $SH = 32$, determine the area of $\triangle FHT$.



(c) In quadrilaterl $KLMN$, KM is perpendicular to LN at R. Medians KP and KQ are drawn in $\triangle KLM$ and $\triangle KMN$ respectively, as shown. If $LR = 6$, $RN = 12, KR = x, RM = 2 + 2$ and the area of $KMPQ$ is 63, determine the value of x.



Problem 3

(a) If $n + 5$ is an even integer, state whether the integer n is even or odd.

(b) If c and d are integers, explain why $cd(c + d)$ is always an even integer.

(c) Determine the number of ordered pairs (e, f) of positive integers where
- $e < f$
- $e + f$ is odd, and
- $ef = 300$

(d) Determine the number of ordered pairs (m, n) of positive integers such that $(m + 1)(2n + m) = 9000$.