Grade 6 Math Circles
October 6, 2021
Irrational Numbers - Problem Set

1. For each of the following numbers, state if they are rational, irrational, or neither. Explain the reasoning behind your choice.

   (a) 7
   (b) $\dfrac{\pi}{2}$
   (c) 17.181818...
   (d) $\sqrt{16}$
   (e) 0

2. State 3 differences between irrational numbers and rational numbers.

3. Compute the following. Express your answer as a whole number or a fraction.

   (a) $3 + \dfrac{2}{5}$

   (b) $\dfrac{1}{\left(\dfrac{3}{11}\right)}$

   (c) $\dfrac{1}{\left(\dfrac{1}{12345}\right)}$

   (d) $2 + \dfrac{1}{6 + \dfrac{1}{3}}$

4. A number's continued fraction expansion is shown below.

$$3 + \cfrac{1}{1 + \cfrac{1}{3 + \cfrac{1}{3 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{2}}}}}}$$

   (a) Is this number rational or irrational?
   (b) What is this number, in improper fraction form?
   (c) What is this number, in decimal form?

5. Find the 5th convergent rational approximation for the golden ratio ϕ. What do you notice? Can you predict what the 10th convergent rational approximation is? You may express your answer in the shortened notation.

6. Research some of the most famous irrational numbers: π, e, and the golden ratio ϕ. How are these numbers defined? How were they discovered? What are some interesting properties? Can you find how they are used in the real world?

7. Consider two rational numbers a and b. If the continued fraction representation of the two numbers are

$$a = [a_0; a_1, a_2, a_3, \ldots, a_n]$$
$$b = [0; a_0, a_1, a_2, a_3, \ldots, a_n],$$

what can you say about the relationship of the two numbers?

8. How would you express the number -1.17 as a continued fraction? Is there more than one way?