Grade 6 Math Circles

October 13, 2021

Divisibility, Factors, and the GCF - Problem Set

1. True or False?
   (a) 16 is divisible by 4.
   (b) 15 divides 5.
   (c) The GCF of 2 and 191 is 1.
   (d) The Euclidean Algorithm always terminates.
   (e) Let x, y, and z be three positive integers. If x divides y and x also divides z, then x is called the GCF of y and z.
2. Find the prime factorization of 144 using the factor tree method. Express your answer in exponential notation.
3. Janet told John that she found a positive integer with 5 factors. John told her that it's impossible, since all factors must come in pairs. Who is right?
4. For the integers 128, 76, and 59, find the their factorizations, the number of factors they each have, and list their factors.
5. Find the GCF of the following pairs of integers using the Euclidean algorithm.
   (a) 328 and 128
   (b) 1072 and 1184
   (c) 5 and 201
   (d) 17 and 85
6. Statement: let a and b be two positive integers with $a < b$. The GCF of a and b is equal to a. Is this statement always true, sometimes true, or never true?
7. Let a and b be two positive integers. If a is a factor of b and b is also a factor of a, what is the relationship between a and b?
8. Let a, b, and c be three positive integers. If a is a factor of b, and b is a factor of c, is a a factor of c?
9. A math teacher wants to split the grade 6 students into equal groups (assuming that they want more than one group). If there are 150 grade 6 students, how many ways can they make the groups?

10. (Challenge Question) Tracy is arranging flower vases for a dinner party. She bought 48 hyacinths, 24 tulips, and 16 azaleas to arrange into vases, and she wants every vase to look identical. If she uses all the flowers she bought, what's the greatest number of vases she can arrange? How many of each type of flowers is in any given vase? *Hint: the GCF of three positive integers a b and c is equal to the GCF of (the GCF of a and b) and c.*

11. (Challenge Question) Given that the GCF of of 252 and 105 is 21, what is the GCF of 105 and 42? You may not explicitly compute the GCF of 105 and 42. *Hint: use the fact that* $42 = 252 - (105 \times 2)$ *and the steps in the Euclidean algorithm!*