



Grade 7/8 Math Circles

October 6th, 2021

Probability - Problem Set

1. A 6-sided die is rolled, where each side has a number between 1 and 6.
 - (a) What is the probability of rolling a 2?
 - (b) What is the probability of rolling a 6?
 - (c) What is the probability of rolling a 1 or 5?
 - (d) What is the probability of rolling a number greater than or equal to 3?
2. There are 50 monkeys in an exhibit at the zoo. Of these 50 monkeys, 24 are male and the remaining are females. Suppose we were to randomly pick a monkey, with the event A = the monkey is a male.
 - (a) Determine $P(A)$.
 - (b) What is \bar{A} and $|\bar{A}|$?
 - (c) Use any method to determine $P(\bar{A})$.
3. There are two types of birds in a bird sanctuary: Blue Jays and Cardinals. Let the event A = the bird is a Cardinal, with $|A| = 42$ and $P(A) = 0.6$.
 - (a) What is \bar{A} and $P(\bar{A})$?
 - (b) Determine S .
 - (c) Use any method to determine $|\bar{A}|$.
4. Suppose we have two independent events A and B , where $P(A) = 0.35$ and $P(B) = 0.8$. Determine $P(A \cap B)$.
5. Suppose we have two events A and B , where $P(A \cap B) = 0.3$ and $P(B) = 0.75$. Determine $P(A | B)$.
6. For two events A and B , we have $P(A) = 0.5$, $P(B) = 0.6$, and $P(A \cap B) = 0.3$.
 - (a) Determine $P(A | B)$.
 - (b) Determine $P(B | A)$.
 - (c) Using the results from parts (a) and (b), are A and B independent or dependent?



7. In a sack of marbles, the marbles are categorized by the following attributes that have no influence on one another: the marbles are either *transparent* or **opaque**; and the marbles are either red or blue. The events are A = the marble is **opaque**, and B = the marble is red. There are a total of 125 marbles in the sack, with $P(A) = 0.56$ and $P(\bar{B}) = 0.4$.
- (a) Determine S , $|A|$, $|\bar{A}|$, $|B|$, and $|\bar{B}|$.
 - (b) Are the events A and B independent or dependent? Explain.
 - (c) Determine $P(A \cap B)$, $P(A \cap \bar{B})$, $P(\bar{A} \cap B)$, and $P(\bar{A} \cap \bar{B})$.
 - (d) What is the sum of the probabilities from part (c)? Why is this the case?
8. Use the Number of Pairings formula to determine how many pairings there are for the given number of people.
- (a) 1 person
 - (b) 2 people
 - (c) 10 people
 - (d) 50 people
 - (e) 100 people
9. Use the general formula for the Birthday Problem to determine the probability that at least 2 people have the same birthday for the given number of people. Round to 4 decimal places.
- (a) 3 people
 - (b) 5 people
 - (c) 8 people
 - (d) 10 people

Bonus Questions

10. A new backpack is released that comes in 1 of 50 different colours. Every student in a school orders exactly one of these backpacks online. Unfortunately, they are not given a choice of colour and the colour of the backpack is picked at random, where each of the 50 colours have the same likelihood of being picked. Suppose that for a given number of students, we want to find the probability for the event A = at least 2 people have the same coloured backpack.
- (a) Using a similar method to the Birthday Problem, find a general formula for $P(A)$ where n is the number of people.
 - (b) Determine $P(A)$ for $n = 10$. Round to 4 decimal places.



(c) For what value of n is $P(A) = 0.45$ approximately?

11. Determine the general formula for the Birthday Problem in the case of a leap year (366 days in a year).