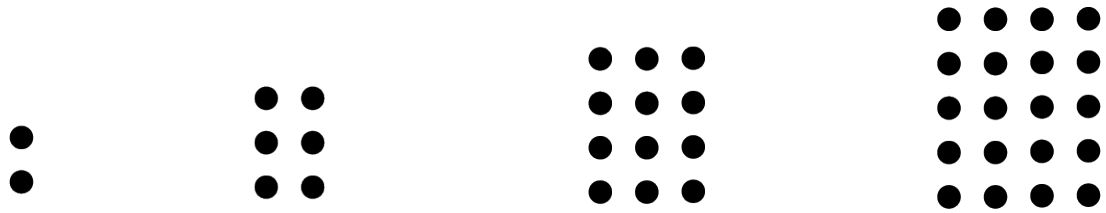
Grade 7/8 Math Circles

November 3, 2021

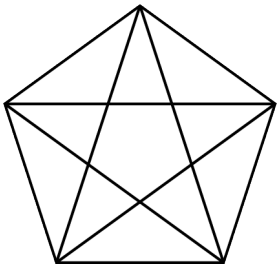
Polygonal Numbers - Problem Set

1. Recall that the n^{th} triangular number can be found using the formula $\frac{n^2+n}{2}$. Compute the 10^{th}, 17^{th}, and the 56^{th} triangular number.
2. Compute the following.
   (a) The 6^{th} term of the sequence of hexagonal numbers.
   (b) The 9^{th} term of the sequence of octagonal numbers.
   (c) The 5^{th} term of the sequence of icosagonal numbers (20 sides).
3. Find a closed-form formula for the n^{th} term of the sequence of
   (a) Tridecagonal numbers (13 sides)
   (b) Enneadecagonal numbers (19 sides)
   (c) Icositetragonal numbers (24 sides)
4. An *oblong* number is the number of dots in a rectangular grid with one more row than column. The first four oblong numbers are 2, 6, 12, and 20, as shown below.



   What is the 9^{th} oblong number?
5. An n-gon is an n-sided regular polygon. For example, a 5-gon is a pentagon.

   A diagonal of a polygon is a line segment drawn between any two vertices (corners). For example, a pentagon has 5 diagonals, shown on the image below.



How many diagonals does an n-gon have? *(Hint: try sketching out the first few n-gons starting from the triangle and counting the number of diagonals.)*

6. Compute the first 10 triangular numbers using the closed-form formula for the sequence of triangular numbers. Then, compute the first 5 hexagonal numbers by using the generalized closed-form formula. What do you notice about these two sequences?

7. Without using a calculator, find the sum of the integers from 1 to 100. In other words, find

$$1 + 2 + 3 + ... + 100$$

Use a similar strategy to find

$$18 + 19 + 20 + ... + 77 + 79$$

8. Using square numbers, compute the sum of the first 15 consecutive positive even integers. Then, find a general closed-form formula for the sum of the first n consecutive positive even integers.