

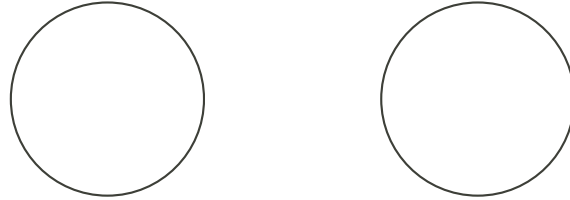


Intermediate Math Circles

Wednesday Nov 3 2021

Geometry II: Angles and Circles

What do we know about circles?



- _____
- _____
- _____
- _____
- Definition: A circle is _____

- Definition: A chord is _____

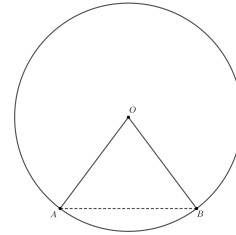
- Definition: A diameter is _____



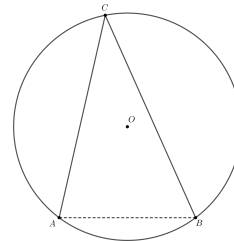
We are going to take a look at a number of theorems related to circles.

We will give some more definitions, then introduce some of the theorems.

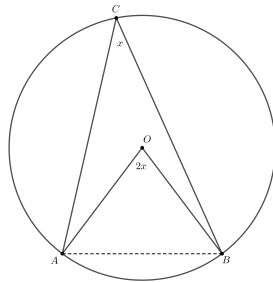
Definition: A central angle is



Definition: An inscribed angle is



Circle Theorem 1: The central angle subtended by a chord is twice the angle of an inscribed angle subtended by the same chord.

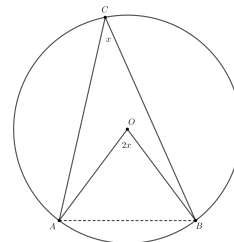


Proof of Circle Theorem 1.

There are two cases we need to look at:

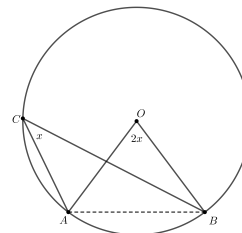
Case 1: The centre of the circle is in the inscribed angle.

We will prove this case.



Case 2: The centre of the circle is outside the inscribed angle.

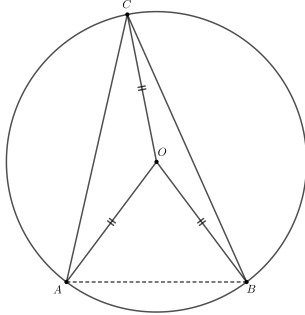
The proof will be asked as a question in the problem set.



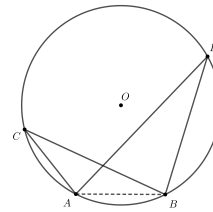
Proof of Circle Theorem 1.

Case 1: The centre of the circle is in the inscribed angle.

Join C to O.

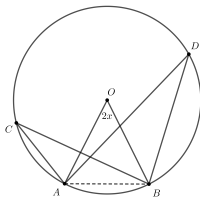


Circle Theorem 2: Two inscribed angles subtended by the same chord and on the same side of the chord are equal. This means for the following diagram $\angle ACB = \angle ADB$.



Proof of Circle Theorem 2

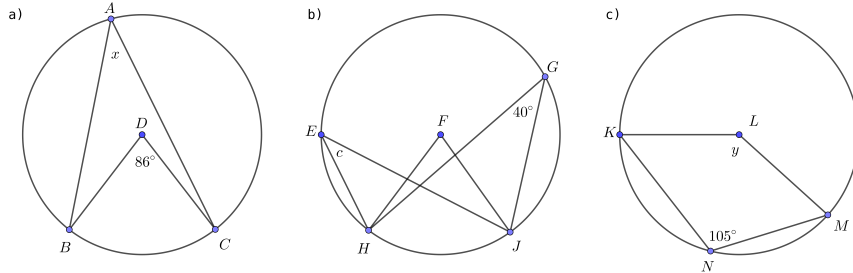
We will draw central angle subtended from chord AB . We will let $\angle AOB = 2x$.





Exercises:

For each question, find the value of the unknowns. Justify your answers.

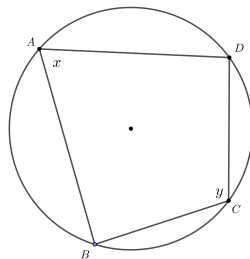


Circle Theorem 3: An inscribed angle subtended by a diameter is a right angle. In the diagram AB is a diameter and, therefore, $\angle ACB = 90^\circ$.

Proof of Circle Theorem 3:

Cyclic Quadrilaterals:

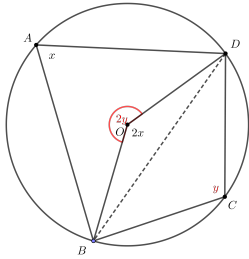
A quadrilateral that has all its vertices lying on the same circle is called a *cyclic quadrilateral*. In our diagram, $ABCD$ is a cyclic quadrilateral.



Circle Theorem 4: The opposite angles of a cyclic quadrilateral are supplementary. In the diagram, $x + y = 180^\circ$

Proof of Circle Theorem 4:

Construct radii BO, DO and chord BD .



Exercises 2:

For each question, find the value of the unknowns. Justify your answers.

