Grade 9/10 Math Circles
November 24, 2021
Complex Numbers Lesson 2 - Solutions

Solutions

1. Show that if $z = a + bi \neq 0$ then $z^{-1}z = 1$.

$$\begin{aligned} z^{-1}z &= \frac{a-bi}{a^2+b^2} \times \frac{a+bi}{1} \\ &= \frac{a^2+b^2}{a^2+b^2} \\ &= 1 \end{aligned}$$

as required.

2. Given $z_1 = 4 + 3i$ and $z_2 = 5 - i$

(a) Determine z_2^{-1}

$$\begin{aligned} z_2^{-1} &= \frac{\overline{5-i}}{|5-i|^2} \\ &= \frac{5+i}{5^2+(-1)^2} \\ &= \frac{5+i}{26} \end{aligned}$$

(b) Determine $z_1 z_2^{-1}$

$$\begin{aligned} z_1 z_2^{-1} &= \frac{4+3i}{1} \times \frac{5+i}{26} \\ &= \frac{20+4i+15i+3i^2}{26} \\ &= \frac{20+4i+15i-3}{26} \\ &= \frac{17+19i}{26} \end{aligned}$$

(c) Determine $z_1 \div z_2$

$$\begin{aligned} z_1 \div z_2 &= \frac{4+3i}{5-i} \\ &= \frac{4+3i}{5-i} \times \frac{5+i}{5+i} \\ &= \frac{20+4i+15i+3i^2}{5^2-i^2} \\ &= \frac{20+4i+15i-3}{5+1} \\ &= \frac{17+19i}{26} \end{aligned}$$

(d) Explain why your answers for (b) and (c) are the same.

They are the same because the division $z_1 z_2^{-1}$ is defined as the multiplication z_2^{-1}.

3. Evaluate each of the following.

(a) $(5-7i)+(6+5i) = 11-2i$

(b) $(3+4i)\times(11-9i) = 69+17i$

(c) $(3+2i)-(7-4i) = -4+6i$

(d) $(7-2i)\div(3+i) = \frac{19-13i}{10}$

4. Evaluate each of the following by expanding fully then using powers of i to simplify.

(a) $(1+2i)^3$

$$\begin{aligned} (1+2i)^3 &= (1+2i)(1+2i)(1+2i) \\ &= (1+2i)(1+4i+4i^2) \\ &= 1+4i+4i^2+2i+8i^2+8i^3 \\ &= 1+6i+12i^2+8i^3 \\ &= 1+6i+12(-1)+8(-i) \\ &= -11-2i \end{aligned}$$

(b) $(2-i)^4$

$$\begin{aligned}
(2-i)^4 &= (2-i)(2-i)(2-i)(2-i)\\
&= (2-i)(2-i)(4-4i+i^2)\\
&= (2-i)(8-8i+2i^2-4i+4i^2-i^3)\\
&= (2-i)(8-12i+6i^2-i^3)\\
&= 16-24i+12i^2-2i^3-8i+12i^2-6i^3+i^4\\
&= 16-32i+24i^2-8i^3+i^4\\
&= 16-32i+24(-1)-8(-i)+1\\
&= -7-24i
\end{aligned}$$