Grade 9/10 Math Circles
November 17, 2021
Complex Numbers Lesson 1 - Solutions

Solutions

1. (Referenced as Section 2.5 (1 - 7) in lesson) Solve each equation for $x \in \mathbb{I}$.

   (a) $x^2 = -1$

   (b) $x^2 = -81$

   (c) $x^2 = -10$

   (d) $x^2 + 4 = 0$

   (e) $-x^2 - 9 = 0$

   (f) $\dfrac{x^2+5}{4} = -11$

   (g) $x^2 - 6x + 13 = 0$

   Solution:

   (a) $x = \pm i$

   (b) $x = \pm 9i$

   (c) $x = \pm i\sqrt{10}$

   (d) $x = \pm 2i$

   (e) $x = \pm 3i$

   (f) $x = \pm 7i$

   (g) $x = 3 \pm 2i$

2. (Referenced as Section 3.3 (1 - 7) in lesson) Solve each equation for $x \in \mathbb{C}$. Simplify your answer as much as possible.

   (a) $x^2 + 7 = 0$

   (b) $x^2 + 5x + 15 = 0$

   (c) $x^2 + x - 5 = 0$

   (d) $x^2 = 3x - 4$

(e) $2x^2 - 3x + 7 = 0$

(f) $7x^2 - 4x + 2 = 0$

(g) $-2x^2 - 5x - 10 = 0$

Solution:

(a) $x^2 + 7 = 0$

$$\begin{aligned} x^2 &= -7 \\ x &= \pm i\sqrt{7} \end{aligned}$$

(b) $x^2 + 5x + 15 = 0$

Using quadratic formula, $a = 1$, $b = 5$, $c = 15$. So

$$\begin{aligned} x &= \frac{-5 \pm \sqrt{5^2 - 4(1)(15)}}{2(1)} \\ &= \frac{-5 \pm \sqrt{-35}}{2} \end{aligned}$$

which has negative discriminant, so

$$x = \frac{5 \pm i\sqrt{35}}{2}$$

(c) $x^2 + x - 5 = 0$

Using quadratic formula, $a = 1$, $b = 1$, $c = -5$. So

$$\begin{aligned} x &= \frac{-1 \pm \sqrt{1^2 - 4(1)(-5)}}{2(1)} \\ &= \frac{-1 \pm \sqrt{21}}{2} \end{aligned}$$

(d) $x^2 = 3x - 4$

Rearrange:

$$x^2 - 3x + 4 = 0$$

Using quadratic formula, $a = 1$, $b = -3$, $c = 4$. So

$$\begin{aligned} x &= \frac{3 \pm \sqrt{(-3)^2 - 4(1)(4)}}{2(1)} \\ &= \frac{3 \pm \sqrt{-7}}{2} \end{aligned}$$

which has negative discriminant, so

$$x = \frac{3 \pm i\sqrt{7}}{2}$$

(e) $2x^2 - 3x + 7 = 0$

Using quadratic formula, $a = 2$, $b = -3$, $c = 7$. So

$$\begin{aligned} x &= \frac{3 \pm \sqrt{(-3)^2 - 4(2)(7)}}{2(2)} \\ &= \frac{3 \pm \sqrt{-47}}{4} \end{aligned}$$

which has negative discriminant, so

$$x = \frac{3 \pm i\sqrt{47}}{4}$$

(f) $7x^2 - 4x + 2 = 0$

Using quadratic formula, $a = 7$, $b = -4$, $c = 2$. So

$$\begin{aligned} x &= \frac{4 \pm \sqrt{(-4)^2 - 4(7)(2)}}{2(7)} \\ &= \frac{4 \pm \sqrt{-40}}{14} \end{aligned}$$

which has negative discriminant, so

$$\begin{aligned} x &= \frac{4 \pm i\sqrt{40}}{14} \\ &= \frac{4 \pm 2i\sqrt{10}}{14} \\ &= \frac{2 \pm i\sqrt{10}}{7} \end{aligned}$$

(g) $-2x^2 - 5x - 10 = 0$

Using quadratic formula, $a = -2$, $b = -5$, $c = -10$. So

$$\begin{aligned} x &= \frac{5 \pm \sqrt{(-5)^2 - 4(-2)(-10)}}{2(-2)} \\ &= \frac{5 \pm \sqrt{-55}}{-4} \end{aligned}$$

which has negative discriminant, so

$$\begin{aligned} x &= \frac{5 \pm i\sqrt{55}}{-4} \\ &= \frac{-5 \pm i\sqrt{55}}{4} \end{aligned}$$