Intermediate Math Circles
October 27, 2021
Geometry I: Geometry and Angles

Getting Started

In geometry, there are certain terms that we just understand. For example, if you were asked to define an angle, you might struggle with appropriate words but could easily draw an angle. Try writing a definition.

What follows is a short list of definitions that will be useful in our discussions.

Angle Related Definitions

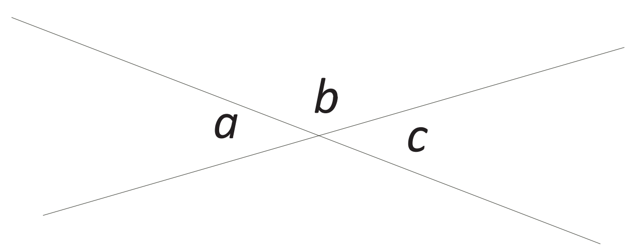
A/an ______________________________ is any angle measuring between $0°$ and $90°$.
A/an ______________________________ is an angle measuring exactly $90°$.
A/an ______________________________ is any angle measuring between $90°$ and $180°$.
A/an ______________________________ is an angle measuring exactly $180°$.

Two angles whose sum is $180°$ are called ___________________________________.
Two angles whose sum is $90°$ are called ___________________________________.

A/an ______________________________ triangle has three sides of different length.
A/an ______________________________ triangle has two sides of equal length.
A/an ______________________________ triangle has three sides of equal length.

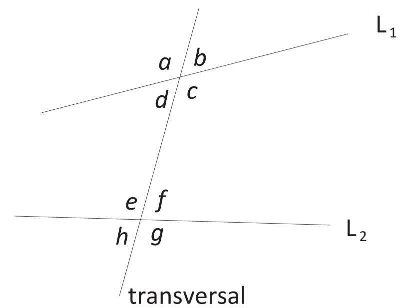
When two lines intersect, four angles are formed. The angles that are directly opposite to each other are called ____.

Prove: Opposite angles are equal.



Proof:

A *transversal* is any line that intersects two (or more) lines at different points.



Two angles located on the same side of the transversal between the two lines are called ______________________________.

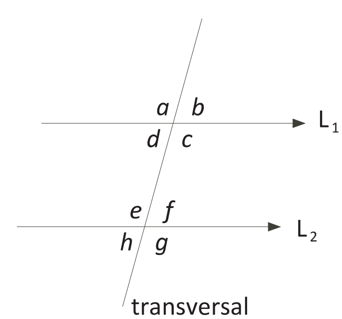
Two non-adjacent angles located on opposite sides of the transversal between the two lines are called ____.

Two angles each of which is on the same side of one of two lines cut by a transversal and on the same side of the transversal are called ____.

A/an ______________________________ is a logical statement which is assumed to be true. We just accept the truth of the statement.

Axiom:

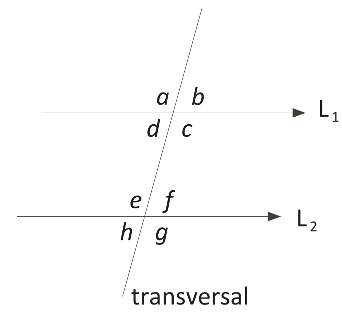
If a transversal cuts two parallel lines, then the co-interior angles are supplementary. That is, the two co-interior angles add to 180°.



Another axiom that we could have started with is: "the angles in a triangle sum to 180°." As a result of starting with the first axiom, we will be able to prove the second axiom (it will not be an axiom for us).

Prove:

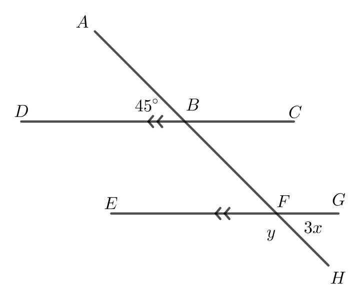
If a transversal cuts two parallel lines, then the alternate angles are equal and the corresponding angles are equal.



Proof:

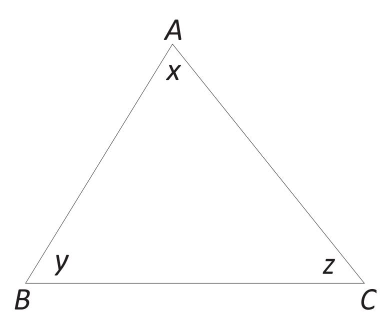
Example:

1) Given $DC \parallel EG$ and AH is a transversal meet the parallel lines at B and F. Also given $\angle ABD = 45°$. Find the value of x and y.



Prove:

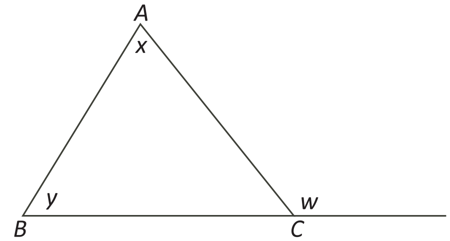
In any triangle, the sum of the interior angles is $180°$.



Proof:

Definition:

A/an ______________________________ is the angle between one side of a triangle and the extension of an adjacent side.



Prove:

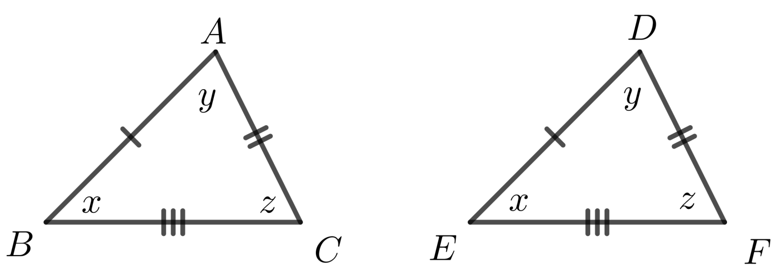
An exterior angle of a triangle is equal to the sum of the opposite interior angles.

Proof:

Congruent Triangles:

Two triangles are said to be congruent if their corresponding sides are equal and their corresponding angles are equal.

For the following $\triangle ABC$ and $\triangle DEF$

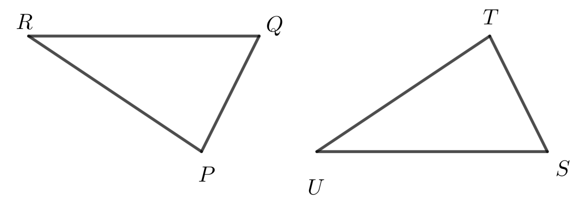


Since, we are given:

$$AB = DE$$
$$BC = EF$$
$$AC = DF$$
$$\angle ABC = \angle DEF$$
$$\angle BAC = \angle EDF$$
$$\angle ACB = \angle DFE$$

Then we say that $\triangle ABC$ is congruent to $\triangle DEF$. We write this as $\triangle ABC \cong \triangle DEF$.

Conversely, if $\triangle PQR \cong \triangle TSU$ (notice the order of the letters) then the following facts are true.



Congruency Postulates:

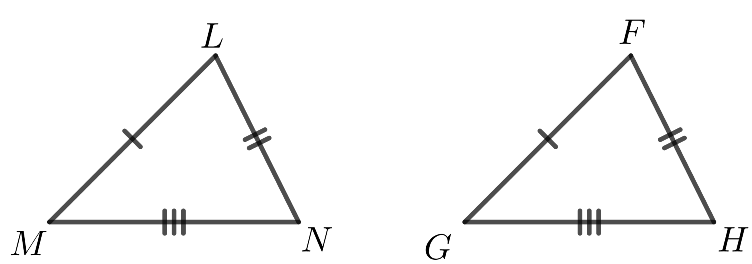
We do not need all 6 equalities to show that two triangles are congruent.

A *postulate* is a logical statement which is assumed to be true. We just accept the truth of the statement. This is similar to an axiom.

There are three triangle congruency postulate that we will use.

1) The Side-Side-Side Postulate (SSS) states that if the three sides of one triangle are congruent to three sides of another triangle, then the two triangles are congruent.

Given the following diagram.

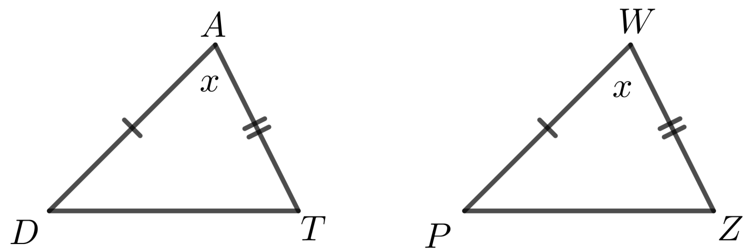


We can state that $\triangle LMN \cong \triangle FGH$ because of the Side-Side-Side Postulate (SSS). Here is a proof:

Note: Once we prove a triangle with a congruency postulate, we get the other three equalities. In this case we now know that:

$$\angle LMN = \angle FGH \text{(corresponding angles of congruent triangles)}$$
$$\angle MNL = \angle GHF \text{(corresponding angles of congruent triangles)}$$
$$\angle NLM = \angle HFG \text{(corresponding angles of congruent triangles)}$$

2) The Side-Angle-Side Postulate (SAS) states that if two sides and the included angle of one triangle are congruent to two sides and the included angle of another triangle, then the two triangles are congruent.



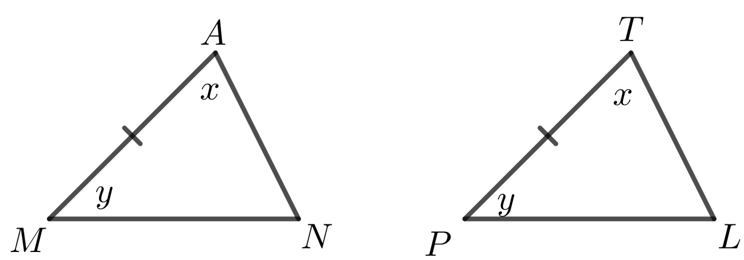
Given the following diagram.

We can state that $\triangle TDA \cong \triangle ZPW$ because of the Side-Angle-Side Postulate (SAS). Here is a proof:

Note: Once we prove a triangle with a congruency postulate, we get the other three equalities. In this case we now know that:

$$\begin{aligned} \angle ADT &= \angle WPZ (\text{corresponding angles of congruent triangles }) \\ \angle DTA &= \angle PZW (\text{corresponding angles of congruent triangles }) \\ DT &= PZ (\text{corresponding sides of congruent triangles }) \end{aligned}$$

3) The Angle-Side-Angle Postulate (ASA) states that if two angles and the included side of one triangle are congruent to two angles and the included side of another triangle, then the two triangles are congruent.



Given the following diagram.

We can state that $\triangle MAN \cong \triangle PTL$ because of the Angle-Side-Angle Postulate (ASA). Here is a proof:

Note: Once we prove a triangle with a congruency postulate, we get the other three equalities. In this case we now know that:

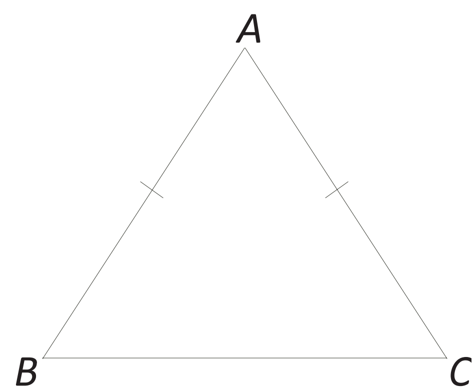
$$\angle MNA = \angle PLT \text{(corresponding angles of congruent triangles)}$$
$$AN = TL \text{(corresponding sides of congruent triangles)}$$
$$MN = PL \text{(corresponding sides of congruent triangles)}$$

And Even One More Proof:

We are going to use the Side-Angle-Side Postulate to prove the following.

Prove:

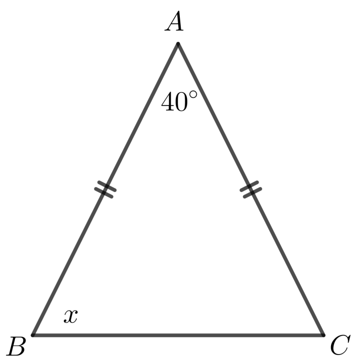
If a triangle is isosceles, then the angles opposite the equal sides are equal.



Proof:

Two more examples;

1) Find the value of x.



2) Find the value of y.