Intermediate Math Circles

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Geometry I: Geometry and Angles

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Getting Started - Basic Definitions

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Definition: An angle is the space formed by two lines or rays diverging from a common point (the vertex). The space is commonly measured in degrees.

On the next page is a list of terms that we will use in this week's Math Circle.

Getting Started - Basic Definitions

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A scalene triangle has three sides of different length.

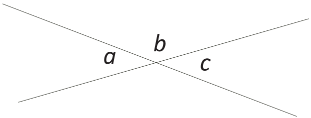
An isosceles triangle has two sides of equal length.

An equilateral triangle has three sides of equal length.

When two lines intersect four angles are formed. The angles that are directly opposite to each other are called opposite angles.

Are Opposite Angles equal, really?

Prove: Opposite angles are equal.



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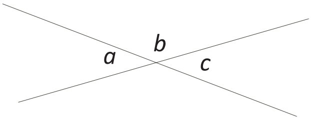
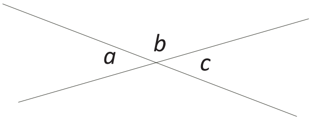


Proof:

We want to show that $a = c$.

Prove: Opposite angles are equal.



Proof:

We want to show that $a = c$.

Since a and b form a straight angle, $a + b = 180^\circ$. (1)

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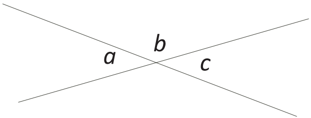

Proof:

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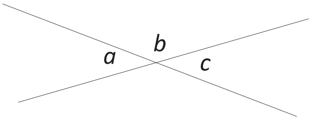

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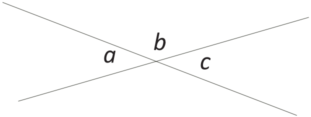

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b is common to both sides so $a = c$ follows.

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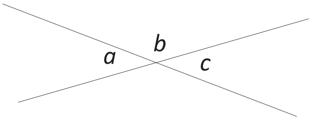

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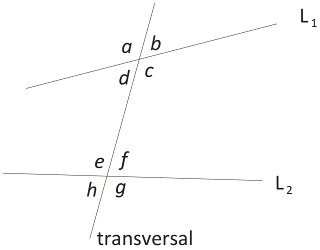
Therefore, opposite angles are equal.

More Terminology

A *transversal* is any line that intersects two (or more) lines at different points.



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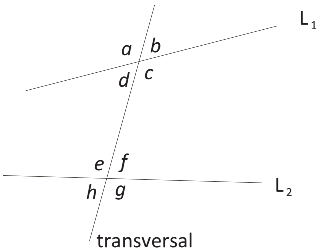

Over the next few pages, we will define three types of angles related to transversals.

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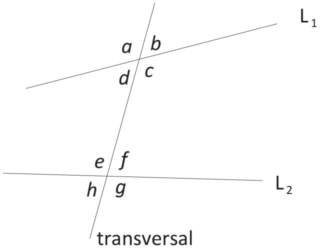
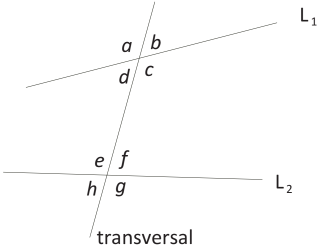


Over the next few pages, we will define three types of angles related to transversals.

For each definition, state all possible angle pairs that satisfy the definition.

More Terminology

Two angles located on the same side of the transversal between the two lines are called $\underline{co - interior\ angles}$.



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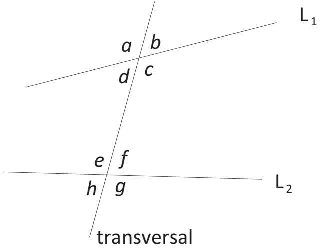
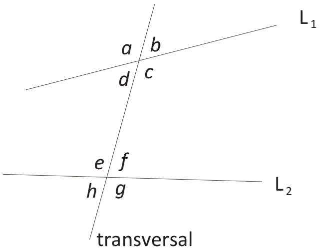


$\angle d$ and $\angle e$ are co-interior angles.

$\angle c$ and $\angle f$ are co-interior angles.

More Terminology

Two non-adjacent angles located on opposite sides of the transversal between the two lines are called *alternate (interior) angles*.



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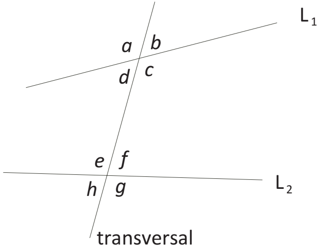
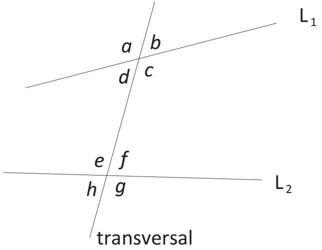


$\angle d$ and $\angle f$ are alternate interior angles.

$\angle c$ and $\angle e$ are alternate interior angles.

More Terminology

Two angles each of which is on the same side of one of two lines cut by a transversal and on the same side of the transversal are called *corresponding angles*.



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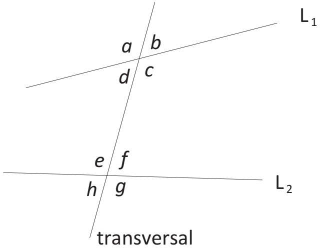

$\angle a$ and $\angle e$ are corresponding angles.

$\angle d$ and $\angle h$ are corresponding angles.

$\angle b$ and $\angle f$ are corresponding angles.

$\angle c$ and $\angle g$ are corresponding angles.

Where Should We Begin?

An *axiom* is a logical statement which is assumed to be true. We just accept the truth of the statement.

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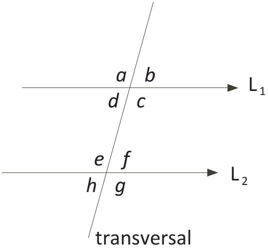
Axiom:

If a transversal cuts two parallel lines, then the co-interior angles are supplementary. That is, the two co-interior angles add to 180°.



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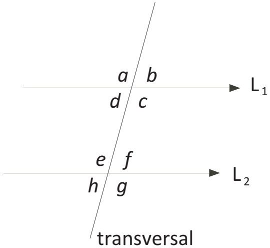
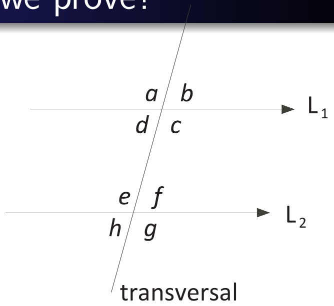


Another axiom that we could have started with is: "the angles in a triangle sum to 180°." As a result of starting with the first axiom, we will be able to prove the second axiom (it will not be an axiom for us).

Prove:

If a transversal cuts two parallel lines, then the alternate angles are equal and the corresponding angles are equal.



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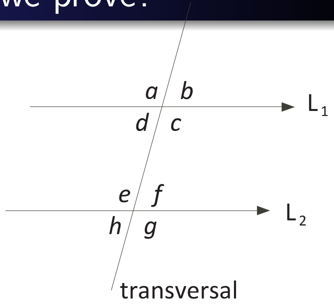
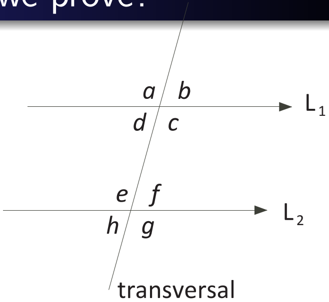


Proof:
We will show that two alternate interior angles d and f are equal.



Prove:

If a transversal cuts two parallel lines, then the alternate angles are equal and the corresponding angles are equal.

Proof:

We will show that two alternate interior angles d and f are equal.
Since d and c form a straight line, $d + c = 180^\circ$. (1)

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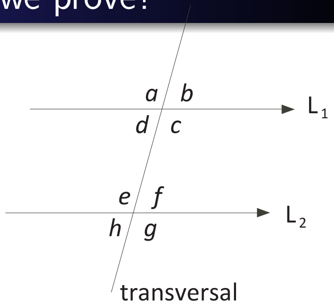

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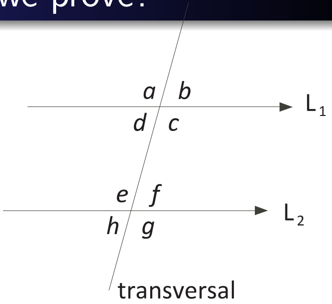

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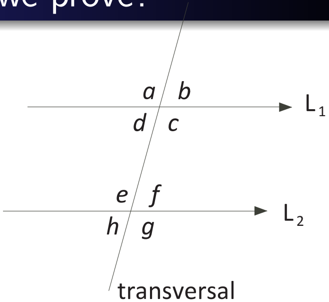

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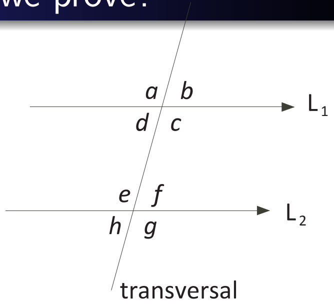
c is common to both sides so $d = f$ follows.



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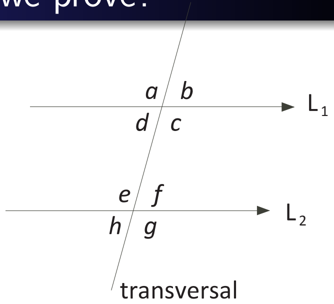

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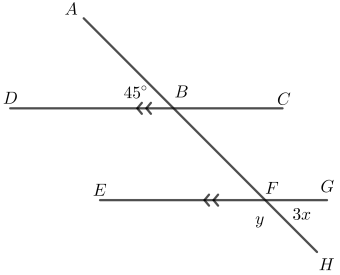
Therefore, alternate interior angles are equal.

The proof of the equality of corresponding angles is very similar to the proof presented here and, thus, will not provided.

An Example

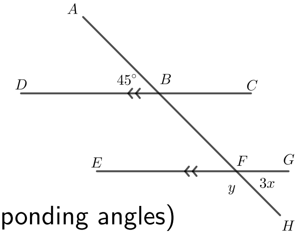
1) Given $DC \parallel EG$ and AH is a transversal that meets the parallel lines at B and F. Also given is $\angle ABD = 45°$. Find the value of x and y.



Take a few minutes to answer the above.

An Example

Here is one possible solution. (There are a few other similar solutions that we will not show.)



$\angle ABD = \angle BFE$ (corresponding angles)

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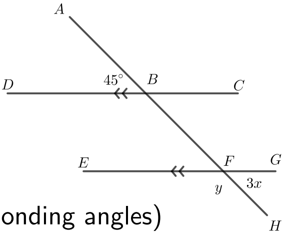

$\angle ABD = \angle BFE$ (corresponding angles)
Therefore, $\angle BFE = 45°$.

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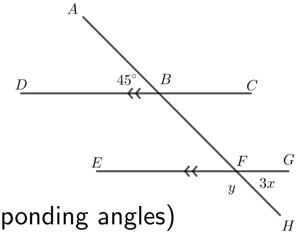

$\angle ABD = \angle BFE$ (corresponding angles)
Therefore, $\angle BFE = 45°$.
$\angle GFH = \angle BFE$ (opposite angles)

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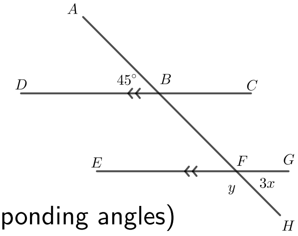

$\angle ABD = \angle BFE$ (corresponding angles)
Therefore, $\angle BFE = 45^\circ$.
$\angle GFH = \angle BFE$ (opposite angles)
Therefore, $3x = 45^\circ$ and $x = 15^\circ$

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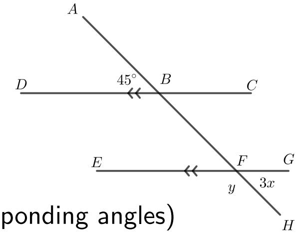

$\angle ABD = \angle BFE$ (corresponding angles)
Therefore, $\angle BFE = 45°$.
$\angle GFH = \angle BFE$ (opposite angles)
Therefore, $3x = 45°$ and $x = 15°$
$y + 3x = 180°$ (straight angle)

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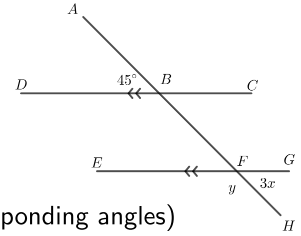
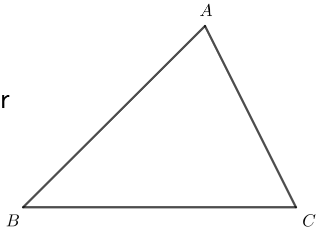


$\angle ABD = \angle BFE$ (corresponding angles)
Therefore, $\angle BFE = 45^\circ$.
$\angle GFH = \angle BFE$ (opposite angles)
Therefore, $3x = 45^\circ$ and $x = 15^\circ$
$y + 3x = 180^\circ$ (straight angle)
Therefore, $y = 135^\circ$.

As a result of a proven result, what can we prove?

Prove:

In any triangle, the sum of the interior angles is 180°.



Proof: See next page

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Proof:

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Prove:

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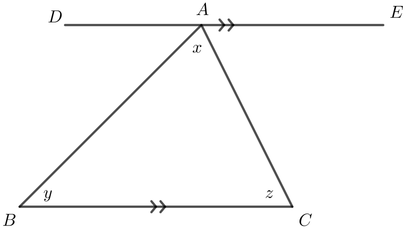
Proof:

Construct $\triangle ABC$.
Let $\angle BAC = x$, $\angle ABC = y$, and $\angle ACB = z$.
Through A draw line segment DE parallel to BC.



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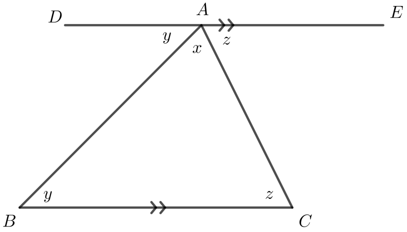
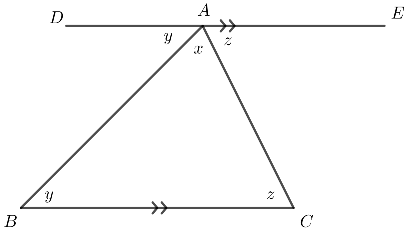


Since $DE \parallel BC$, $\angle DAB = \angle ABC = y$ (alternate angles) and $\angle EAC = \angle ACB = z$ (alternate angles).

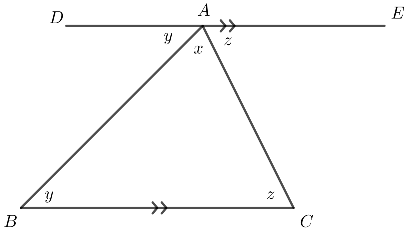
Proof (continued):



Since $DE \parallel BC$, $\angle DAB = \angle ABC = y$ (alternate angles) and $\angle EAC = \angle ACB = z$ (alternate angles).

$$\begin{aligned} \angle ABC + \angle BAC + \angle ACB &= y + x + z \\ &= \angle DAB + \angle BAC + \angle EAC \\ &= 180^\circ \qquad \text{a straight angle.} \end{aligned}$$

Proof (continued):



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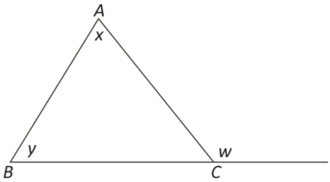
Therefore, the angles in a triangle sum to 180°.

One More Proof

Definition:

An *exterior angle* is the angle between one side of a triangle and the extension of an adjacent side. In the diagram, w is an exterior angle.



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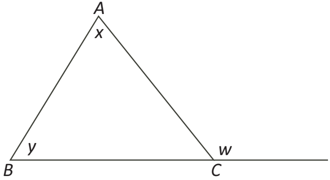

Prove:

An exterior angle of a triangle is equal to the sum of the opposite interior angles. (In the above diagram, x and y are the opposite interior angles related to w.)

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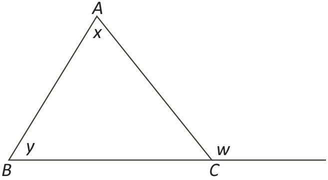

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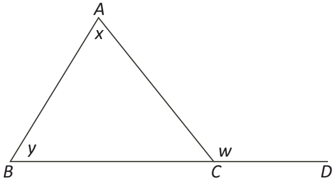
Proof:

See next page

One More Proof

Prove:

An exterior angle of a triangle is equal to the sum of the opposite interior angles. In the diagram, x and y are the opposite interior angles related to w. (i.e. Show $w = x + y$)



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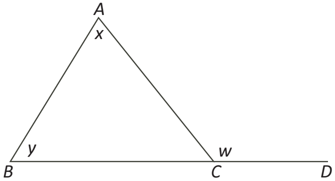

Proof:

Since BCD is a straight angle, $\angle BCA + w = 180^\circ$. (1)

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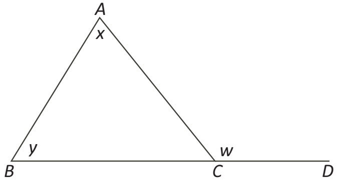


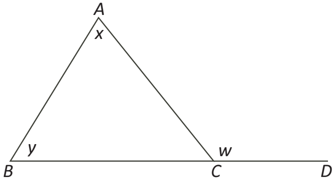
Proof:

Since BCD is a straight angle, $\angle BCA + w = 180^\circ$. (1)
In $\triangle ABC$, $\angle BCA + x + y = 180^\circ$. (2)

One More Proof

Prove:

An exterior angle of a triangle is equal to the sum of the opposite interior angles. In the diagram, x and y are the opposite interior angles related to w. (i.e. Show $w = x + y$)



Proof:

Since BCD is a straight angle, $\angle BCA + w = 180^\circ$. (1)
In $\triangle ABC$, $\angle BCA + x + y = 180^\circ$. (2)
Since $180^\circ = 180^\circ$ in (1) and (2), $\angle BCA + w = \angle BCA + x + y$.
$\angle BCA$ is common to both sides.

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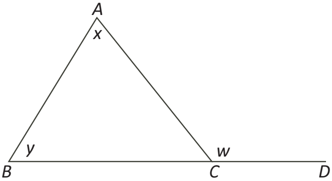

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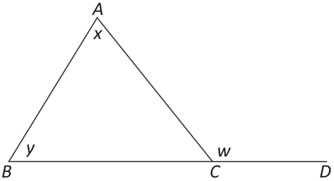

Proof:

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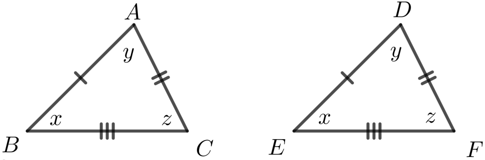
In $\triangle ABC$, $\angle BCA + x + y = 180°$. (2)
Since $180° = 180°$ in (1) and (2), $\angle BCA + w = \angle BCA + x + y$.
$\angle BCA$ is common to both sides.
It follows that $w = x + y$.

Therefore, an exterior angle of a triangle is equal to the sum of the opposite interior angles.

Congruent Triangles

Two triangles are said to be congruent if their corresponding sides are equal and their corresponding angles are equal.
For the following $\triangle ABC$ and $\triangle DEF$

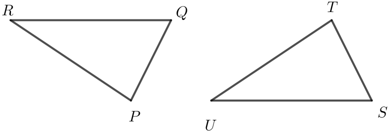


Since, we are given:

$$AB = DE$$
$$BC = EF$$
$$AC = DF$$
$$\angle ABC = \angle DEF$$
$$\angle BAC = \angle EDF$$
$$\angle ACB = \angle DFE$$

Then we say that $\triangle ABC$ is congruent to $\triangle DEF$. We write this as $\triangle ABC \cong \triangle DEF$.

Conversely, if $\triangle PQR \cong \triangle TSU$ (notice the order of the letters) then the following facts are true.



$$PQ = TS$$
$$QR = SU$$
$$PR = TU$$
$$\angle PQR = \angle TSU$$
$$\angle QRP = \angle SUT$$
$$\angle RPQ = \angle UTS$$

Congruent Triangles

We do not need all 6 equalities to show that two triangles are congruent.

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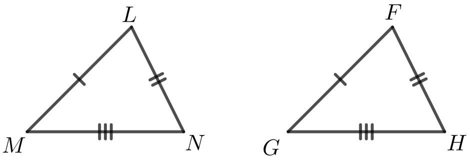
There are three triangle congruency postulates that we will use.

Congruent Postulate SSS

1) The Side-Side-Side Postulate (SSS) states that if the three sides of one triangle are congruent to three sides of another triangle, then the two triangles are congruent.

Given the following diagram.



We can state that $\triangle LMN \cong \triangle FGH$ because of the Side-Side-Side Postulate (SSS). Here is a proof:

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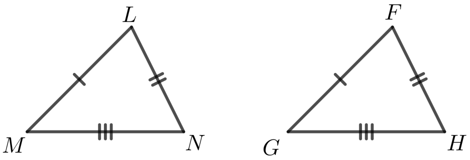


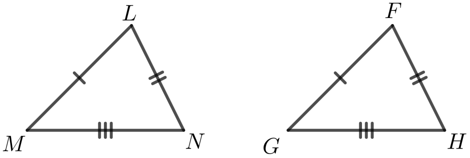
We can state that $\triangle LMN \cong \triangle FGH$ because of the Side-Side-Side Postulate (SSS). Here is a proof:

$\triangle LMN \cong \triangle FGH$ (SSS)

$$LM = FG(\text{given})$$
$$MN = GH(\text{given})$$
$$NL = HF(\text{given})$$

Congruent Postulates SSS

Given the following diagram.



From the previous page we know $\triangle LMN \cong \triangle FGH$ (SSS)

Note: Once we prove a triangle with a congruency postulate, we get the other three equalities. In this case we now know that:

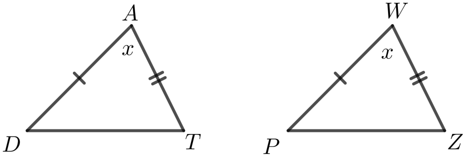
$\angle LMN = \angle FGH$(corresponding angles of congruent triangles)

$\angle MNL = \angle GHF$(corresponding angles of congruent triangles)

$\angle NLM = \angle HFG$(corresponding angles of congruent triangles)

2) The Side-Angle-Side Postulate (SAS) states that if two sides and the included angle of one triangle are congruent to two sides and the included angle of another triangle, then the two triangles are congruent.

Given the following diagram.



We can state that $\triangle TDA \cong \triangle ZPW$ because of the Side-Angle-Side Postulate (SAS). Here is a proof:

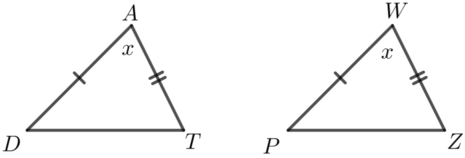
$\triangle TDA \cong \triangle ZPW$ (SAS)

$$AD = WP \text{(given)}$$

$$\angle DAT = \angle PWZ \text{(given)}$$

$$AT = WZ \text{(given)}$$

Given the following diagram.



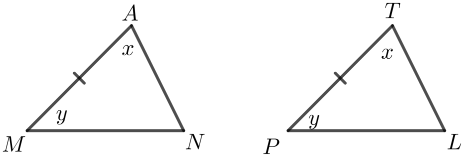
From the previous page we know $\triangle TDA \cong \triangle ZPW$ (SAS)

Note: Once we prove a triangle with a congruency postulate, we get the other three equalities. In this case we now know that:

$$\angle ADT = \angle WPZ \text{(corresponding angles of congruent triangles)}$$
$$\angle DTA = \angle PZW \text{(corresponding angles of congruent triangles)}$$
$$DT = PZ \text{(corresponding sides of congruent triangles)}$$

3) The Angle-Side-Angle Postulate (ASA) states that if two angles and the included side of one triangle are congruent to two angles and the included side of another triangle, then the two triangles are congruent.

Given the following diagram.



We can state that $\triangle MAN \cong \triangle PTL$ because of the Angle-Side-Angle Postulate (ASA). Here is a proof:

$\triangle MAN \cong \triangle PTL$ (ASA)

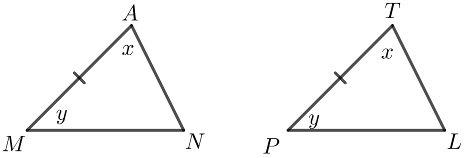
$$\angle MAN = \angle PTL (\text{given})$$

$$MA = PT (\text{given})$$

$$\angle AMN = \angle TPL (\text{given})$$

Congruent Postulates ASA

Given the following diagram.



From the previous page we know $\triangle MAN \cong \triangle PTL$ (ASA)

Note: Once we prove a triangle with a congruency postulate, we get the other three equalities. In this case we now know that:

$$\angle MNA = \angle PLT \text{(corresponding angles of congruent triangles)}$$
$$AN = TL \text{(corresponding sides of congruent triangles)}$$
$$MN = PL \text{(corresponding sides of congruent triangles)}$$

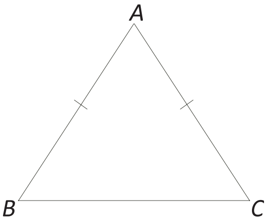
And Even One More Proof

We are going to use the Side-Angle-Side Postulate to prove the following.

Prove:

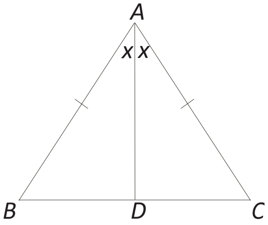
If a triangle is isosceles, then the angles opposite the equal sides are equal.

(This is known as the Isosceles Triangle Theorem.)



Proof:

Construct isosceles $\triangle ABC$ so that $AB = AC$. We are required to prove $\angle ABC = \angle ACB$.



Construct AD, the angle bisector of $\angle BAC$.

It follows that $\angle BAD = \angle DAC$. (Definition of an angle bisector.)

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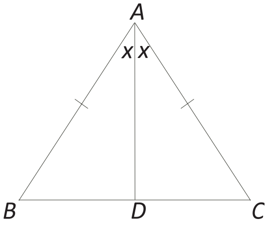


Construct AD, the angle bisector of $\angle BAC$.

It follows that $\angle BAD = \angle DAC$. (Definition of an angle bisector.)
In $\triangle BAD$ and $\triangle CAD$,

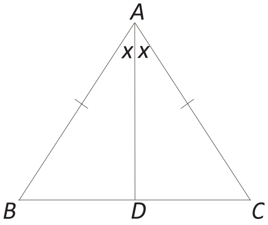
$$AB = AC \text{(given)}$$
$$\angle BAD = \angle CAD \text{(constructed)}$$
$$AD = AD \text{(common)}$$

Therefore, $\triangle BAD \cong \triangle CAD$ (SAS).

Proof:

Construct isosceles $\triangle ABC$ so that $AB = AC$. We are required to prove $\angle ABC = \angle ACB$.

Construct AD, the angle bisector of $\angle BAC$.



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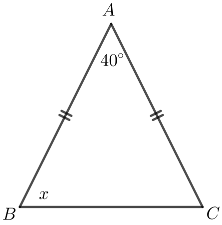
In $\triangle BAD$ and $\triangle CAD$,

$$AB = AC \text{(given)}$$
$$\angle BAD = \angle CAD \text{(constructed)}$$
$$AD = AD \text{(common)}$$

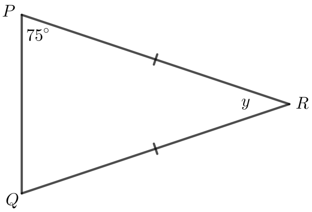
Therefore, $\triangle BAD \cong \triangle CAD$ (SAS).
It follows that $\angle ABC = \angle ACB$, as required.

Two more examples

1) Find the value of x.

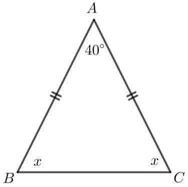


2) Find the value of y.



Solutions

1) Since $\triangle ABC$ is isosceles then $\angle ABC = \angle ACB = x$. The three angles in the triangle sum to 180°, therefore $x + x + 40 = 180$ or $2x = 140$ and $x = 70^\circ$.



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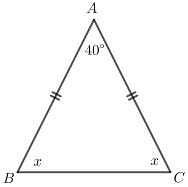
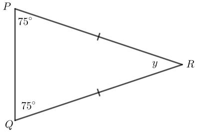


2) Since $\triangle PQR$ is isosceles then $\angle RPQ = \angle RQP = 75$. The three angles in the triangle sum to 180°, therefore $75 + 75 + y = 180$ and $y = 30^\circ$.



Time to Solve Some Problems

You can now do Problem Set 1.