

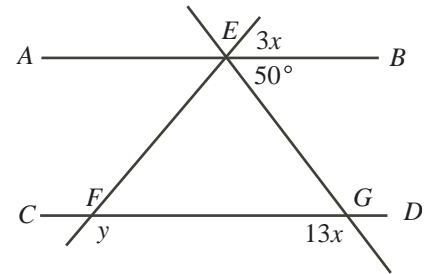


# Intermediate Math Circles

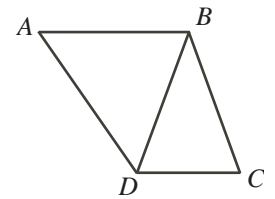
## October 27 2021

### Problem Set 1

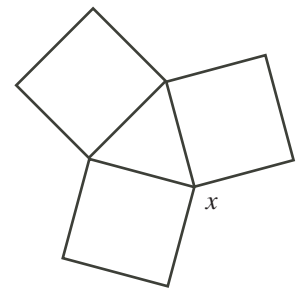
1. In the diagram,  $AB$  is parallel to  $CD$ . Determine the values of  $x$  and  $y$ .



2. In the diagram,  $AB$  is parallel to  $DC$  and  $AB = BD = BC$ . If  $\angle A = 52^\circ$ , determine the measure of  $\angle DBC$ .

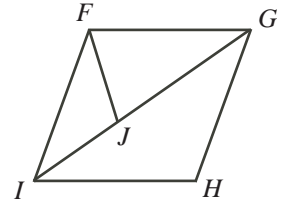


3. The diagram shows three squares of the same size. What is the value of  $x$ ?



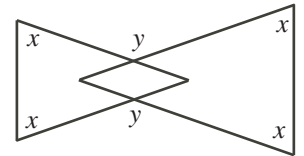


4. The diagram shows a rhombus  $FGHI$  and an isosceles triangle  $FGJ$  in which  $GF = GJ$ . Angle  $FJI$  equals  $111^\circ$ . What is the measure of angle  $JFI$ ?

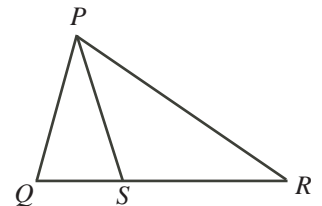


5.  $ABCD$  is a square. The point  $E$  is outside the square so that  $CDE$  is an equilateral triangle. Determine the measure of angle  $BED$ .

6. The diagram shows two isosceles triangles in which the four angles marked  $x$  are equal. The two angles marked  $y$  are also equal. Find an equation relating  $x$  and  $y$ . (Note: The sum of the angles in a quadrilateral is  $360^\circ$ )

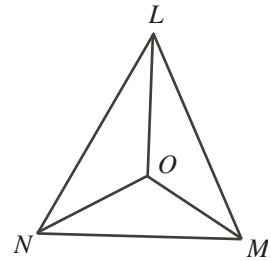


7. In the diagram,  $QSR$  is a straight line.  $\angle QPS = 12^\circ$  and  $PQ = PS = RS$ . What is the measure of  $\angle QPR$ ?

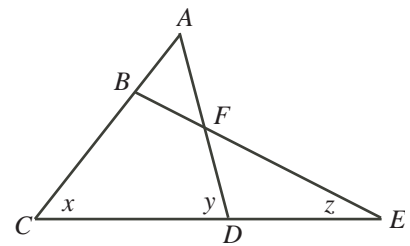




8. The three angle bisectors of triangle  $LMN$  meet at a point  $O$  as shown. Angle  $LMN$  is  $68^\circ$ . What is the size of angle  $LOM$ ?



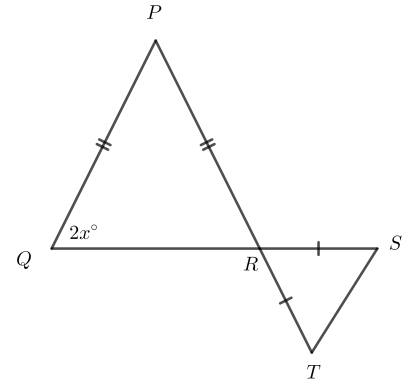
9. In the figure shown,  $AB = AF$  and  $ABC$ ,  $AFD$ ,  $BFE$ , and  $CDE$  are all straight lines. Show that  $x - y + 2z = 0$ .



10. Triangle  $ABC$  has a right angle at  $B$ .  $AC$  is extended to  $D$  so that  $CD = CB$ . The bisector of angle  $A$  meets  $BD$  at  $E$ . Prove that  $\angle AEB = 45^\circ$ .

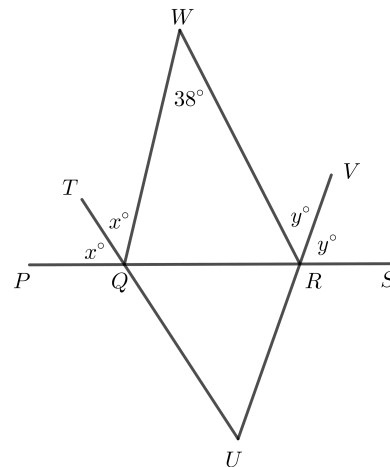


11. In the diagram,  $R$  is the point of intersection of  $PT$  and  $QS$ ,  $PQ = PR$ , and  $RS = RT$ . If  $\angle PQR = 2x^\circ$ , what is the measure of  $\angle RST$ , in terms of  $x$ .



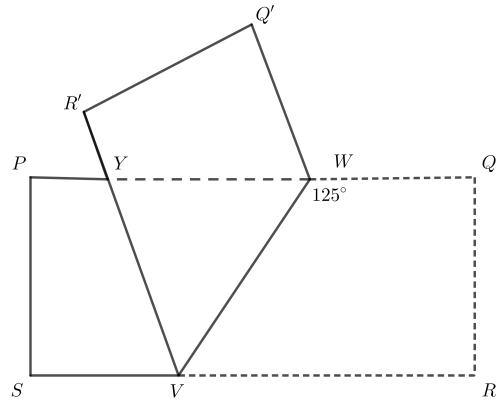
12. In  $\triangle PQR$ ,  $PQ = PR$ .  $PQ$  is extended to  $S$  so that  $QS = QR$ . Prove that  $\angle PRS = 3(\angle QSR)$ .

13. In the diagram, points  $Q$  and  $R$  lie on  $PS$  and  $\angle QWR = 38^\circ$ . If  $\triangle TQP = \triangle TQW = 0^\circ$ ,  $\triangle VRS = \triangle VRW = y^\circ$ , and  $U$  is the point of intersection of  $TQ$  extended and  $VR$  extended, then what is the measure of  $\angle QUR$ ?



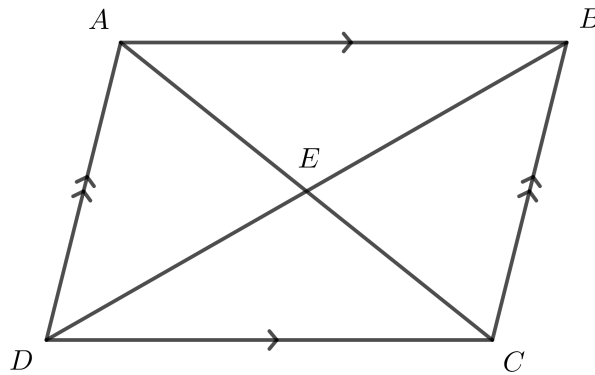


14. In the diagram,  $PQRS$  represents a rectangular piece of paper. The paper is folded along a line  $VW$  so that  $\angle VWQ = 125^\circ$ . When the folded paper is flattened, points  $R$  and  $Q$  have moved to points  $R'$  and  $Q'$  respectively and  $R'V$  crosses  $PW$  at  $Y$ . What is the measure of  $\angle PYV$ ?



15. Prove that the diagonals of a parallelogram bisect each other. (You will need to use a congruent triangle postulate)

That is:



$ABCD$  is a parallelogram. The diagonals  $AC$  and  $BD$  intersect at  $E$ .  
Prove that  $AE = EC$  and  $BE = ED$