Grade 6 Math Circles
February 9th, 2022
Percentages and Applications Solutions

Exercise Solutions

Exercise 1
Change the following percentages to decimals or decimals to percentages.

(a) 55% (b) 9% (c) 0.4 (d) 0.02

Exercise 1 Solution

(a) 0.55 (b) 0.09 (c) 40% (d) 2%

Exercise 2
Represent each of the following percentages as decimals and fractions.

(a) 0.5% (b) 300% (c) 1.07% (d) 0.084%

Exercise 2 Solution

(a) 0.005 and $\frac{5}{1000}$

(b) 3.0 and $\frac{300}{100}$

(c) 0.0107 and $\frac{107}{10\,000}$

(d) 0.00084 and $\frac{84}{100\,000}$

Exercise 3
Practice finding amounts from percentages by calculating the following.

(a) What is 70% of 34?

(b) What is 0.5% of 500?

(c) If Chen bought a pizza with 8 slices, how many slices is 37.5% of the pizza?

(d) If James has \$20 and his brother takes 15% of his money, how much money did James lose?

Exercise 3 Solution

We can use the amount from percentage formula to solve these problems.

(a) 70% of 34 is equal to $34 \times 0.7 = 23.8$.

(b) 0.5% of 500 is equal to $500 \times 0.005 = 2.5$.

(c) 37.5% of the pizza is $8 \times 0.375 = 3$ slices.

(d) James lost $\$20 \times 0.15 = \$3$.

Exercise 4

Practice finding percentages from amounts by calculating the following.

(a) What percentage is $\frac{7}{35}$?

(b) If Julia got 33 marks on her math test that was out of 40 marks, what percent grade did she get?

(c) If Dave and Liu bought 18 of the 120 chocolate bars at the store, what percent of the store's stock did they purchase?

(d) If Anaisha has 16 rings and sells 6 of them, what percent of her rings did she keep?

Exercise 4 Solution

We can use the percentage from amounts formula to solve these problems.

(a) $7 \div 35 \times 100 = 20$ so the percentage is 20%.

(b) $33 \div 40 \times 100 = 82.5$ so Julia got 82.5% on her math test.

(c) $18 \div 120 \times 100 = 15$ so Dave and Liu purchased 15% of the store's stock.

(d) Since Anaisha sold 6 of her 16 rings, that means there are 10 rings that she kept. Since $10 \div 16 \times 100 = 62.5$, that means she kept 62.5% of her rings.

Exercise 5

Practice using discounts by calculating the following.

(a) Hasan's store is having a 15% off sale. What would be the discounted price for an item that is regularly $70.00?

(b) Ashley has been wanting to purchase a new laptop but waits until it's on sale. The laptop she wants is regularly $999.00 and the store has discounted it by 32% so she purchases it with the discount. How much money did Ashley save by waiting to buy the laptop?

(c) Mathew's favourite restaurant gives a 20% off discount to students. If he wants a sandwich and soup combo that's regularly priced at $11.00, what price will the restaurant charge him if he is a University student?

(d) Trisha is buying coffee for her co-workers. The company gets 8% off orders over $30.00. If Trisha orders $34.50 worth of coffee, how much money will she save on her order?

Exercise 5 Solution

Note that you could've used either method from the lesson for finding discounts to calculate the final answers, or alternatively used the amount from percentage formula for parts (b) and (d) as shown below.

(a) The discounted price for the item would be $70 \times 0.85 = \$59.50$.

(b) Ashley saved $999 \times 0.32 = \$319.68$ by waiting to buy the laptop.

(c) The restaurant will charge Mathew $11 \times 0.8 = \$8.80$.

(d) Trisha will save $34.50 \times 0.08 = \$2.76$ on her order.

Exercise 6

Practice using sales tax by calculating the following.

(a) Jacqueline wants to buy a shirt that is $15.00. How much money will she actually spend at the register when HST is added to her bill?

(b) Elena is buying ice cream for her and her three friends. If ice cream cones are $3 each and HST is added to her bill, how much money will Elena end up spending?

(c) Miguel is buying a couch for his new apartment that is priced $1250.00 at the store. How much more will he have to pay for HST?

Exercise 6 Solution

(a) Jacqueline will spend $15 \times 1.13 = \$16.95$ at the register with HST added to her bill.

(b) Since Elena is buying ice cream for her and her three friends, she will be buying four cones in total. This means that the ice creams alone cost $12, and therefore Elena will end up spending $12 \times 1.13 = \$13.56$ when HST is added to her bill.

(c) We simply have to calculate the amount of tax, so we can use the equation for calculating amount from percentage. Miguel will have to pay an additional $1250 \times 0.13 = \$162.50$ for HST.

Exercise 7

Practice finding interest and accumulated value by calculating the following.

(a) Fran arranges to borrow $15\,200$ from a company with an interest rate of 7%. If she takes 18 months to pay back the money, how much interest will she end up paying?

(b) Celine saves $32\,000$ in her savings account at the bank with an interest rate of 3%. What will be the accumulated value of her bank account in 10 years?

Exercise 7 Solution

(a) We can use the formula for simple interest. In this case, $P = \$15\,200$, $r = 0.07$, and $t = \frac{18}{12} = 1.5$. Therefore, $I = \$15\,200 \times 0.07 \times 1.5 = \$1596$. So, Fran will end up paying $\$1596$ of interest.

(b) We can use the formula for accumulated interest. In this case, $P = \$32\,000$, $r = 0.03$, and $t = 10$. Therefore, $A = \$32\,000 \times (1 + 0.03 \times 10) = \$41\,600$. So, the accumulated value of Celine's bank account will be $\$41\,600$.

Exercise 8

Practice finding the present value by calculating the following.

(a) Jack borrowed money 2 years ago and has now repaid an accumulated value of $\$9800$. If the interest rate for his borrowing was 6%, how much money did Jack originally borrow?

(b) Kylie has $\$68\,000$ in her savings account after 210 months of keeping her money there with an interest rate of 4%. How much money must Kylie have put into her savings account?

Exercise 8 Solution

(a) We can use the formula for present value. In this case, $A = \$9800$, $r = 0.06$, and $t = 2$. Therefore, $P = \$9800 \div (1 + 0.06 \times 2) = \$8750$. So, Jack must have originally borrowed $\$8750$.

(b) We can again use the present value formula. In this case, $A = \$68\,000$, $r = 0.04$, and $t = \frac{210}{12} = 17.5$. Therefore, $A = \$68\,000 \div (1 + 0.04 \times 17.5) = \$40\,000$. So, Kylie must have put $\$40\,000$ into her savings account.

Problem Set Solutions

1. For the following:

   - Represent any percentages as a decimal and as a fraction.
   - Represent any decimals as a percentage and as a fraction.
   - Represent any fractions as a percentage and as a decimal.

Also, for the fractions, the numerators shouldn't have decimals and you should use 100, 1000, 10 000 or 100 000 for the denominators, similar to the table from the lesson.

(a) 0.777% (b) 50.05% (c) 1.63 (d) 0.908 (e) $\frac{49}{100}$ (f) $\frac{252}{10\,000}$

> *Solution*:
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> (a) 0.00777 and $\frac{777}{100\,000}$ (c) 163% and $\frac{163}{100}$ (e) 49% and 0.49
>
> (b) 0.5005 and $\frac{5005}{10\,000}$ (d) 90.8% and $\frac{908}{1000}$ (f) 2.52% and 0.0252

2. Tanya and her classmates had a math quiz out of 25 marks. Tanya earned 24 marks, Hilda earned 20 marks, and Joshua earned 17 marks. What are their grades as percentages?

> *Solution*: Using the formula for percentage from amount, we can calculate each person's grade separately. Since $24 \div 25 \times 100 = 96$, Tanya got 96% on the quiz. Then, since $20 \div 25 \times 100 = 80$, Hilda got 80% on the quiz. Lastly, since $17 \div 25 \times 100 = 68$, Joshua got 68% on the quiz.

3. Leona borrows $\$52\,000$ from a company with an interest rate of 5% for 3 years, and her husband borrows $\$10\,000$ from a different company with an interest rate of 8% for 18 months. What will be the accumulated value that Leona and her husband will have to repay?

> *Solution*: We can calculate the accumulated value for Leona and her husband separately using the formula for accumulated value and then add the two answers together for the total amount they'll pay together. For Leona's case, we have $P = \$52\,000$, $r = 0.05$, and $t = 3$, so we can calculate that her accumulated value will be $A = \$52\,000 \times (1 + 0.05 \times 3) = \$59\,800$. Then, for her husband's case, we have $P = \$10\,000$, $r = 0.08$, and $t = \frac{18}{12} = 1.5$, so we can calculate that his accumulated value will be $A = \$10\,000 \times (1 + 0.08 \times 1.5) = \$11\,200$. Therefore, the total accumulated value that Leona and her husband will have to repay together will be $\$59\,800 + \$11\,200 = \$71\,000$.

4. Vanessa is going to repay the money she's borrowing at an interest rate of 6% for 21 months. If the accumulated value she is going to repay is \$29 282.50, what is the present value of the money?

> *Solution*: We can use the present value formula to solve this problem. In Vanessa's case, we have $A = \$29\,282.50$, $r = 0.06$ and $t = \frac{21}{12} = 1.75$, so we can calculate that the present value of Vanessa's money is $P = \$29\,282.50 \div (1 + 0.06 \times 1.75) = \$26\,500$.

5. Ravindra and Hongshu made a pizza together. Ravindra ate $\frac{2}{5}$ of the pizza. Hongshu ate half as much as Ravindra. What percentage of the original pizza was left?

> *Solution*: First, note that Hongshu eating half as much as Ravindra means that Hongshu ate $\frac{2}{5} \div 2 = \frac{1}{5}$ of the pizza. Therefore, together, Hongshu and Ravindra ate $\frac{2}{5} + \frac{1}{5} = \frac{3}{5}$ of the pizza. Therefore, there must be $1 - \frac{3}{5} = \frac{2}{5}$ of the original pizza left. We can use the formula for percentage from amount and calculate that $2 \div 5 \times 100 = 40$, so the answer is that there was 40% of the original pizza left.

6. A bicycle at Store A costs \$450. The regular price of the same bicycle at Store B is 15% more than it is at Store A. The bicycle is on sale at Store B for 10% off of the regular price. What is the sale price of the bicycle at Store B?

> *Solution*: First, we should calculate the regular price of the bicycle at Store B. Since the price is 15% more than at Store A, we should calculate 15% of \$450 and add it to the price. Using our formula for amount from percentage, we calculate that the bicycle at Store B costs $\$450 \times 0.15 = \$67.50$ more than at Store A. Therefore the regular price at Store B is $\$450 + \$67.50 = \$517.50$. (Note that we could do this calculation in a different way, similar to sales tax, where we could've done $\$450 \times 1.15 = \$517.50$).
>
> Next, we want to subtract 10% from this regular price at Store B to get the sale price of the bicycle at Store B. Since $\$517.50 \times 0.1 = \$51.75$, the sale price at Store B is $\$517.50 - \$51.75 = \$465.75$. (Note again that we could've done this calculation differently as mentioned in the lesson. We just used method one for calculating discounts, and instead could've used method two by calculating $\$517.50 \times 0.9 = \$465.75$).

It's important to consider how this answer is different than if we cancelled out the increase of 15% and decrease of 10% and tried to calculate the sale price at Store B by just adding 5% to the price at Store A (which would give the higher answer $450 \times 1.05 = \$472.50$). This happens because 10% of $450 is smaller than 10% of $517.50, and when we do just an increase of 5%, that would be like adding the 15% then subtracting the 10% that's only based on the $450 amount instead of the $517.50 amount. When we instead take 10% off of the $517.50, it ends up subtracting a larger number than 10% of $450, so we get the smaller, correct answer.

7. Each time Jim pours water from a jug into a glass, exactly 10% of the water remaining in the jug is used. If the jug starts out being full of water, what is the minimum number of times that he must pour water into a glass so that less than half the water remains in the jug?

Solution: First, note that we can represent the full jug of water as 100% of the water. Then, we should note that half of the water can be represented as 50% of the water. Next, we can start making calculations. For each pour, we should calculate the amount left from the previous pour minus 10%. Similar to discounts, we can do this by calculate 10% of the remaining water then subtracting it, or we can do this by multiplying by 90% since that's how much will remain. Below, I have used method two to do my calculations:

Number of Pours	Amount of Water Left
1	$1 \times 0.9 = 0.9 = 90\%$
2	$0.9 \times 0.9 = 0.81 = 81\%$
3	$0.81 \times 0.9 = 0.729 = 72.9\%$
4	$0.729 \times 0.9 = 0.6561 = 65.61\%$
5	$0.6561 \times 0.9 = 0.59049 = 59.049\%$
6	$0.59049 \times 0.9 = 0.531441 = 53.1441\%$
7	$0.531441 \times 0.9 = 0.4782969 = 47.82969\%$

As we can see, the first time we have less than half the water left in the jug is after the seventh pour. Therefore, the answer is that Jim must pour the water at least 7 times so that less than half the water remains in the jug.

8. A bakery just changed their menu prices to reflect a special sale that they're having. Cookies went from \$2 each to \$1.50 each, muffins went from \$5.75 each to \$3.45 each, and cakes went from \$43 each to \$28.81 each. What percentage of discount was applied to each of these items?

Solution: We can solve this problem by first finding how much percentage we have left of each price, and then the discount must be the remaining percentage that we're missing. So, with the cookies, we first use the formula for percentages from amount and calculate \$1.50 ÷ \$2 × 100 = 75, which means the cookies are at 75% of their original price. Now, we can conclude that the cookies must be discounted by 100% − 75% = 25%. We do the same thing for the other two treats. For the muffins, we calculate \$3.45 ÷ \$5.75 × 100 = 60, so the muffins are at 60% of their original price. Therefore, the muffins must be discounted by 100% − 60% = 40%. Then, lastly, for the cakes, we calculate \$28.81 ÷ \$43 × 100 = 67, so the cakes are at 67% of their original price. Therefore, the cakes must be discounted by 100% − 67% = 33%.

9. A restaurant bill, including 13% tax but not including a tip, is \$135.60. The guests want to give the server a 15% tip based on the bill before tax. How much tip should the guests pay the server?

Solution: Since the guests want to tip based on the bill *before* tax, we need to calculate what the bill amount is without tax. We do this by reversing the tax calculation, which involves dividing by 1.13 (since to get tax, we can multiply by 1.13 as we saw in the lesson). So, we calculate \$135.60 ÷ 1.13 = \$120, and this means that the bill was \$120 before tax. (We can check our answer by reapplying tax: \$120 × 1.13 = \$135.60).

An important note: We cannot simply calculate 13% of \$135.60 then subtract that to reverse the tax calculation. This is due to the same reasoning as we saw in the bicycle problem, where 13% of the price including tax will be a bigger number than 13% of the price before tax. If you did this calculation, checking your answer by reapplying tax would show you that something went wrong, which is why checking our answer is so important!

Back to the problem, now that we know the bill amount before tax, we can simply use the amount from percentage formula to get our final answer. We calculate that the guests should tip the server \$120 × 0.15 = \$18.

10. On a science test, Janine got 80% of the 10 multiple choice questions correct and 70% of the 30 short answer questions correct. What percentage of the 40 questions on the test did she answer correctly?

   Solution: We can first find out how many multiple choice questions Janice got correct. Using the amount from percentage formula, we calculate that Janice got $10 \times 0.8 = 8$ of the multiple choice questions correct. Next, we can find out how many of the short answer questions she got correct. Using the same formula, we calculate that Janice got $30 \times 0.7 = 21$ of the short answer questions correct. Now, we can add these two amounts together to know how many questions she got correct on the whole test: $8 + 21 = 29$. Finally, our last step is to use the percentage from amount formula to calculate $29 \div 40 \times 100 = 72.5$, which means that Janice correctly answered 72.5% of the 40 questions on the test.

11. In a container at a candy store, there are 200 candies. Of these candies, 90% are red and the rest are blue. After Yehudi eats some of the red candies, 80% of the remaining candies in the container are red. How many red candies did Yehudi eat?

   Solution: We can start by calculating how many of each candy we start with. Using the amount from percentage formula, since $200 \times 0.9 = 180$, we know that the container originally has 180 red candies and 20 blue candies. Since Yehudi only eats the red candy, we will still have 20 blue candies when he's done eating. Furthermore, the 20 blue candies must be 20% of the container after he's done eating, since there is 80% red candies left. Moreover, since $\frac{20}{100} = 20\%$, it must be that there are 100 candies left in the container in total. So, Yehudi must have eaten $200 - 100 = 100$ red candies.