Grade 6 Math Circles
February 23rd, 2022
Circles Solutions

Exercise Solutions

Exercise 1

Calculate the area and circumference of the following. Round to one decimal point.

(a) A circle with a radius of 2cm.

(b) A circle with a diameter of 7cm.

(c) A circle with a radius of 1m.

Exercise 1 Solution

(a) The area of this circle is 12.6cm^2 and the circumference is 12.6cm.

(b) The area of this circle is 38.5cm^2 and the circumference is 22.0cm.

(c) The area of this circle is 3.1m^2 and the circumference is 6.3m.

Exercise 2

Calculate the volume and surface area of the following cylinders. Round to one decimal point.

(a) A cylinder with a radius of 4m and a height of 3m.

(b) A cylinder with a radius of 6cm and a height of 2cm.

Exercise 2 Solution

(a) The volume of this cylinder is 150.7m^3 and the surface area is 175.8m^2.

(b) The volume of this cylinder is 226.1cm^3 and the surface area is 301.4cm^2.

Exercise 3

Calculate the volume and surface area of a cone with a radius of 6cm, a vertical height of 8cm, and a slant height of 10cm. Round to one decimal point.

Exercise 3 Solution

The volume of this cone is 301.4cm^3 and the surface area is 301.4cm^2.

Exercise 4

Calculate the volume and surface area of the following spheres. Round to one decimal point.

(a) A sphere with a radius of 1m.

(b) A sphere with a diameter of 6cm.

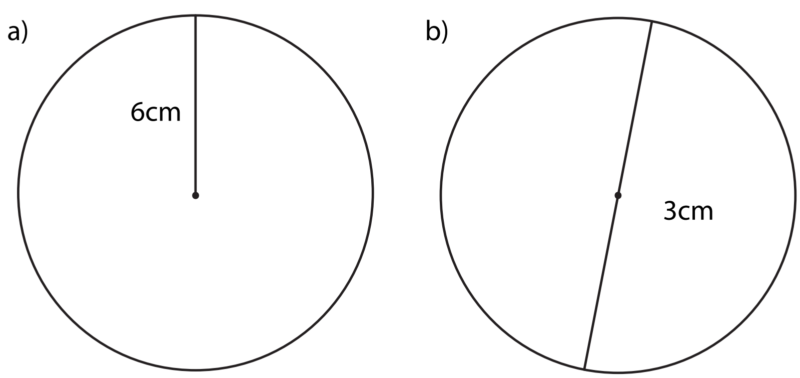
Exercise 4 Solution

(a) The volume of this sphere is 4.2m^3 and the surface area is 12.6m^2.

(b) The volume of this sphere is 113.0cm^3 and the surface area is 113.0cm^2.

Problem Set Solutions

1. Calculate the area and circumference of the circles below. Round to one decimal point.

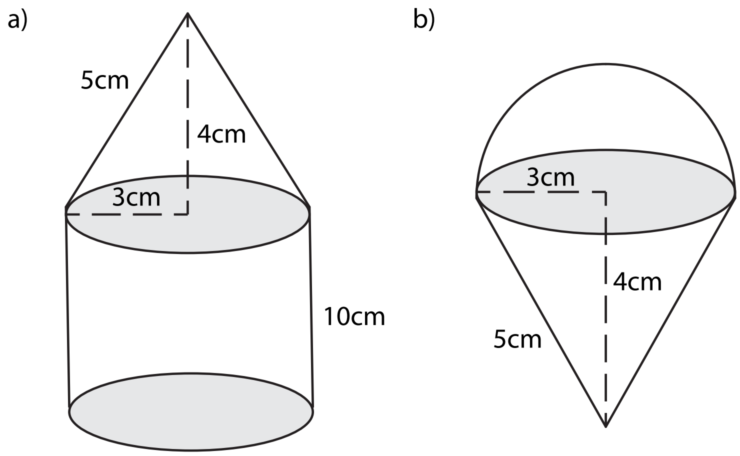


Solution:

a) The area of this circle is 113.0cm^2 and the circumference is 37.7cm.

b) The area of this circle is 7.1cm^2 and the circumference is 9.4cm.

2. Calculate the volume and surface area of the 3D shapes below. Round to one decimal point.



Solution:

a) Note that we can use 3.14 instead of π for calculations since we only need one decimal point of accuracy.

To get the total volume of this 3D shape, we can calculate the volume of the cylinder and the volume of the cone separately, and then add them together.

The volume of the cylinder is 282.6cm^3 and the volume of the cone is 37.68cm^3, therefore the total volume of this 3D shape, rounded to one decimal place, is 320.3cm^3.

To get the total surface area of this 3D shape, we can calculate the surface area of the cylinder without one of the circle bases, and then add the surface area of the cone without the circle base. We do this because those sides of the cylinder and cone are contained inside the bigger 3D shape.

The surface area of the cylinder without one of the circle bases is equal to $\pi r^2 + 2\pi rh = 216.66\text{cm}^2$ and the surface area of the cone without the circle base is $\pi rs = 47.1\text{cm}^2$. Therefore, the total surface area of this 3D shape, rounded to one decimal place, is 263.8cm^2.

b) To get the total volume of this 3D shape, we can calculate the volume of the cone and the volume of the half-sphere separately, and then add them together.

The volume of the cone is 37.68cm^3. To get the volume of the half-sphere, we can calculate the volume of the full sphere multiplied by $\frac{1}{2}$, or divided by 2. So, the volume of the half-sphere is $\frac{1}{2} \times \frac{4}{3}\pi r^3 = 56.52\text{cm}^2$. Therefore, the total volume of this 3D shape, with one decimal place, is 94.2cm.

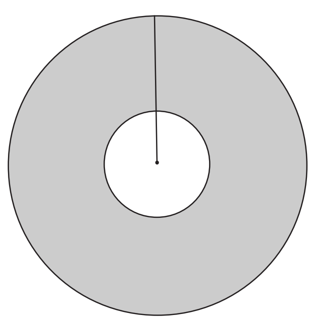
To get the total surface area of this 3D shape, we can calculate the surface area of the cone without the circle base, and then add the surface area of half of a sphere. We do this because the circular base of the cone is contained inside the bigger 3D shape, and we have half of the exterior of the sphere included on the exterior of the 3D shape.

The surface area of the cone without the circle base is $\pi rs = 47.1\text{cm}^2$ and the surface area of the half sphere is $\frac{1}{2} \times 4\pi r^2 = 2\pi r^2 = 56.52\text{cm}^2$. Therefore, the total surface area of this 3D shape, rounded to one decimal place, is 103.6cm^2.

3. We learned that π is a special number related to circles. Research where the number π comes from and explain it in your own words.

Solution: Below is an example of what you could have said:
"The special number π comes from the ratio between the circumference and the diameter of all circles. In other words, for all circles, it's true that $\frac{\text{Circumference}}{\text{Diameter}} = \pi$."

4. In the diagram below, both circles have the same centre. If the radius of the outer circle is 9cm and the radius of the inner circle is $\frac{1}{3}$ of that, what is the area of the shaded region? Furthermore, what percentage of the larger circle is shaded? Round to one decimal point for both answers.

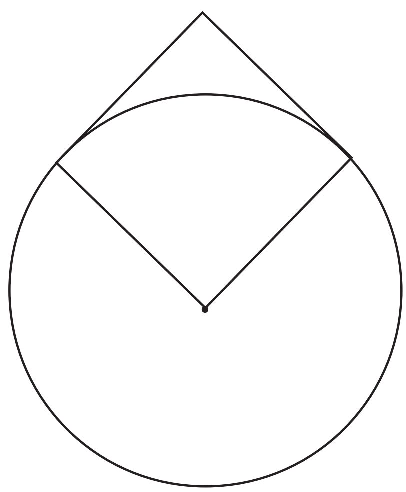


Solution: First, note that the radius of the inner circle must be 3cm. Also note that we can use 3.14 instead of π for calculations since we only need one decimal point of accuracy. We can find the area of the shaded region by calculating the area of the larger circle and subtracting the area of the smaller circle.

The area of the larger circle is 254.34cm^2 and the area of the smaller circle is 28.26cm^2. Therefore, the area of the shaded region, rounded to one decimal point, is 226.1cm^2.

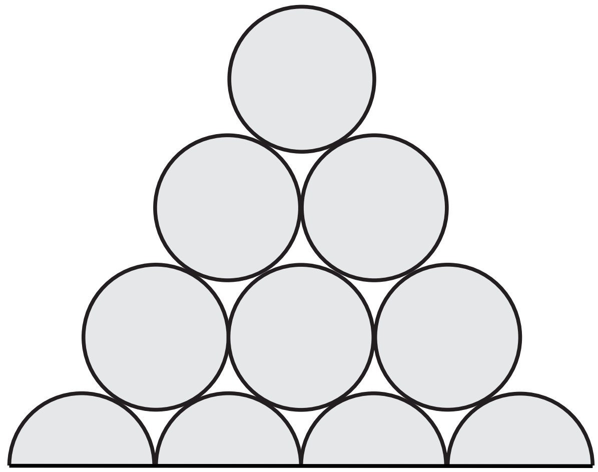
To find the percentage, recall that $(Percentage) = (Part\ of\ amount) \div (Whole\ amount) \times 100$. So, we calculate $226.1\text{cm}^2 \div 254.34\text{cm}^2 \times 100 = 88.89$. Therefore, rounding to one decimal point, approximately 88.9% of the larger circle is shaded.

5. In the diagram below, the square has an area of 9cm^2. What is the area of the circle? Round to one decimal point.



Solution: Note that the area of a square is equal to $(length) \times (width)$ and the sides of squares are equal. So, we can conclude that the sides of the square are 3cm, since $3\text{cm} \times 3\text{cm} = 9\text{cm}^2$. Now, we can see that the square lines up perfectly with the circle so that the bottom corner is at the centre of the circle and each edge attached to that corner goes to the outer edge of the circle. This means those edges are equal to the radius of the circle. So, we know that the radius of the circle is also 3cm. Therefore, using the formula for area of a circle, with 3.14 instead of π, we get that $A = 3.14 \times (3\text{cm})^2 = 3.14 \times 9\text{cm}^2 = 28.26\text{cm}^2$. Therefore, rounding to one decimal point, the area of the circle is 28.3cm^2.

6. In the diagram below, each circle and semi-circle have a radius of 2cm. What is the total area of the shaded regions? Round to one decimal point.

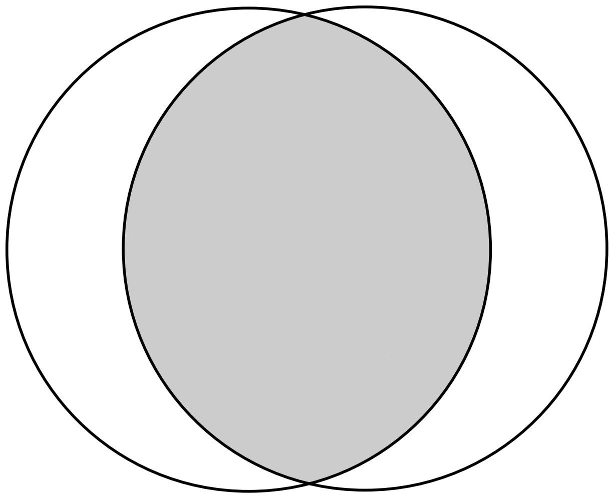


Solution: Using the formula for area of a circle, with $\pi = 3.14$, the area of each full circle will be $A = 3.14 \times (2\text{cm})^2 = 3.14 \times 4\text{cm}^2 = 12.56\text{cm}^2$. Then, since each semi-circle has the same radius, we can simply divide the area of a full circle by 2 to get the area of each semi-circle: $12.56\text{cm}^2 \div 2 = 6.28\text{cm}^2$. Or, alternatively, we could notice that there are four semi-circles, which would make-up the same area as two full circles! Either way, we get that the total area of the semi-circles is 25.12cm^2. Then, we can get that the total area of the full circles is $6 \times 12.56\text{cm}^2 = 75.36\text{cm}^2$. Therefore, adding the area of the semi-circles and full circles together and rounding to one decimal point, we get that the total area of the shaded regions is 100.5cm^2.

7. A circle has an area of $M\text{cm}^2$ and a circumference of $N\text{cm}$. If $\frac{M}{N} = 20$, what is the radius of the circle, in cm?

Solution: We know that the formula for the area of a circle is $A = \pi r^2$ and a formula for circumference is $C = 2\pi r$. Therefore, if the area is $M\text{cm}^2$ and the circumference is $N\text{cm}$, then $\frac{M}{N} = \frac{\pi r^2}{2\pi r}$. We can remove the π and one r from the top and bottom of the fraction since $\frac{\pi}{\pi} = 1$ and $\frac{r}{r} = 1$. So, we are left with $\frac{M}{N} = \frac{r}{2}$. And, since $\frac{M}{N} = 20$, it must be true that $\frac{r}{2} = 20$. In words, this means that when we divide the radius by 2, we get the answer 20cm. Therefore, the radius must be 40cm.

8. The two circles below each have the same radius. The area of the shaded region equals the sum of the areas of the two unshaded regions. If the area of the shaded region is approximately 678.24cm^2, what is the circumference of each circle? Use 3.14 instead of π and round the final answer to one decimal point.



Solution: First, note that since the circles are exactly the same size, their unshaded regions must be equal. Next, we can note that since the area of the shaded region equals the sum of the areas of the two unshaded regions, then the area of the unshaded region of each circle must be $678.24\text{cm}^2 \div 2 = 339.12\text{cm}^2$. Therefore, the area of each full circle is $678.24\text{cm}^2 + 339.12\text{cm}^2 = 1017.36\text{cm}^2$.

Now, since we know that the formula for area of a circle is $A = \pi r^2$, we can find the radius of each circle in order to calculate the circumference later. To do this, we work backwards from the total area. First, we can divide the area by $\pi = 3.14$, which will let us know that $r^2 = 324\text{cm}^2$. Next, we need to find what value of r will make it so that $r \times r = 324\text{cm}^2$. You can do this by trial and error, or by using the square root function on your calculator. Either way, we discover that $r = 18\text{cm}^2$.

Finally, we have the radius of each circle, and we can use it to solve for circumference: $C = 2 \times 3.14 \times 18\text{cm} = 113.04\text{cm}$. Therefore, rounding to one decimal point, our final answer is that the circumference of each circle is 113.0cm.