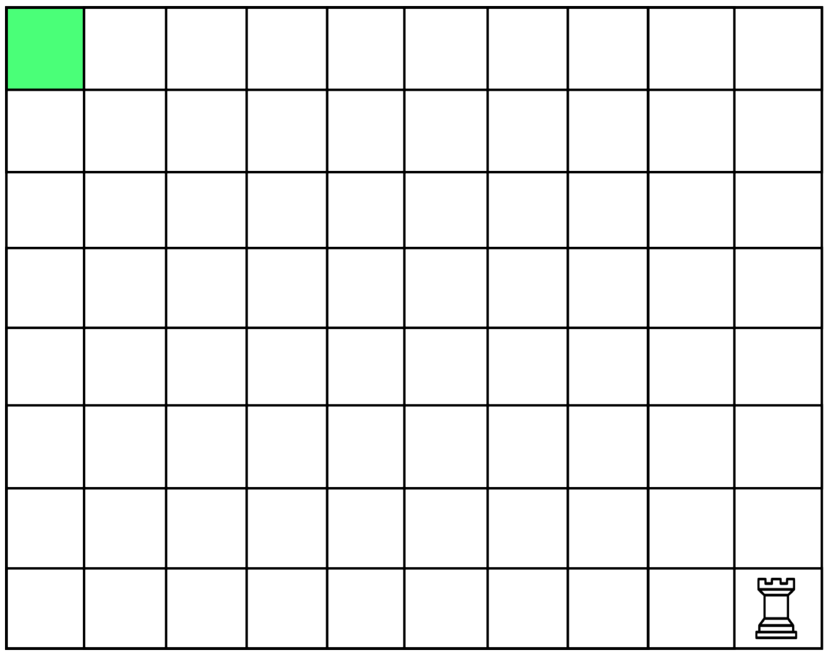
Grade 7/8 Math Circles

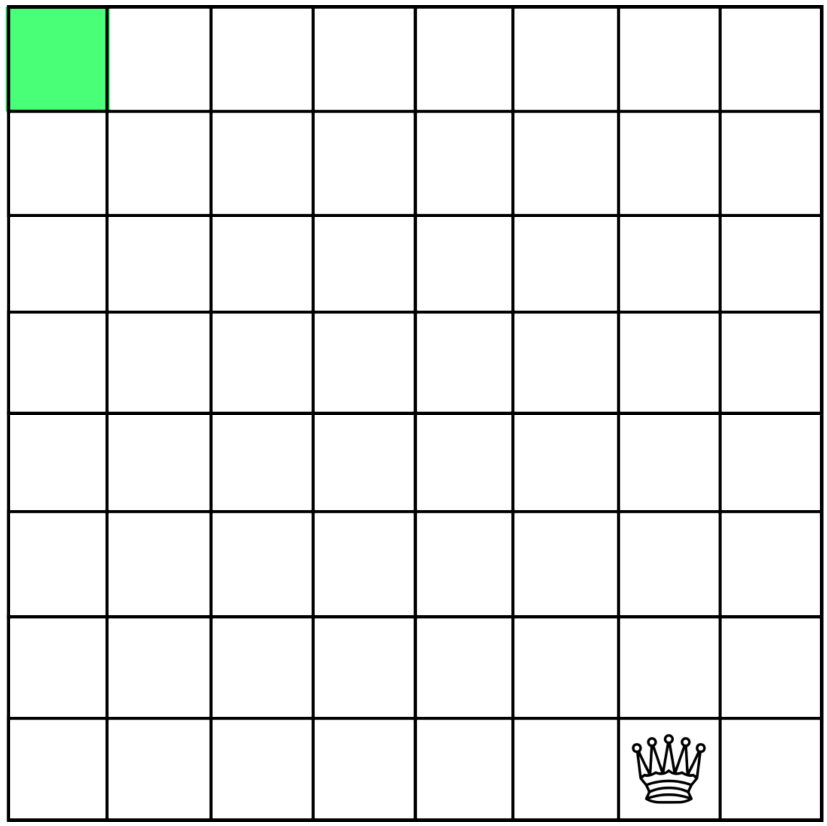
November 24, 2021

Game Theory - Problem Set

1. Suppose the game of rooks is played on a 10×8 board instead of an 8×8 board. What is the winning strategy? Is it better to be player 1 or player 2?



2. Instead of the game of rooks, let us play the game of queens. The game starts off on an 8×8 board with a queen on the square near the bottom-right of the board, as shown below. A queen piece can move in a single direction toward the top-left corner (up, left, or diagonal) for any number of squares in one turn. The objective is to move the queen to the opposite corner of the board from where it started. Using backwards induction, find out if it's better to start the game as player 1 or player 2.



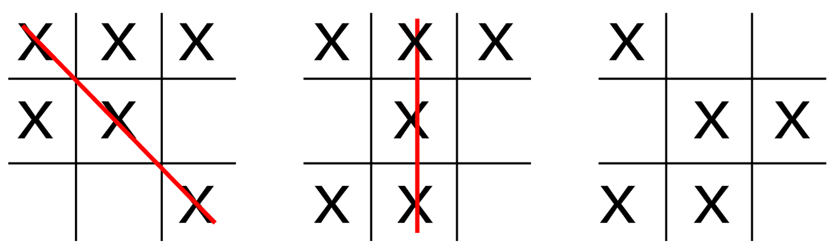
3. This question explores more scenarios of the logic coin game.
   (a) There are 25 coins in a pile. On each turn, a player can remove 1, 2, or 3 coins. The player that takes the last coin **loses.** What is the winning strategy? Is it better to be player 1 or player 2?
   (b) There are 40 coins in a pile. On each turn, a player can remove 2, 3, 4, or 5 coins. The player that takes the last coin **wins.** What is the winning strategy? Is it better to be player 1 or player 2?
   (c) There are 100 coins in a pile. On each turn, a player can remove 2, 3, 4, or 5 coins. The player that takes the last coin **loses.** What is the winning strategy? Is it better to be player 1 or player 2?
4. This question is an extension of the pirate game. Suppose that the rules are exactly the same as the pirate game in the lesson, but the only difference being that the proposer of the coin distribution plan does not have a casting vote. If the voting is a tie, then the proposer of that plan is thrown overboard.

   Suppose you are pirate A, the most senior of the five pirates. What coin distribution plan would you propose to split the 100 gold coins?

5. The calendar game is a 2-player turn based game where players take turns writing down dates. The first player must begin with writing down January 1$^{\text{st}}$. After this, the second player takes the previous date and may increase either the *month* or the *day*, but not both. For example, the next player can choose January 20$^{\text{th}}$ or June 1$^{\text{st}}$, but not March 29$^{\text{th}}$. The player who writes down December 31$^{\text{st}}$ wins. What is the winning strategy?

6. Notakto is another 2-player turn-based game played on a finite number of 3×3 boards. It is a variation of tic-tac-toe, and both players play the same piece on the board (such as an "X"). Each player takes turns placing an X on the board(s) in a vacant space. If one player makes a three-in-a-row on a board, that board becomes "dead" and no more pieces can be played on that board. When one player makes a three-in-a-row and there are no more boards to play on, that player loses.

   For example, if the current state of the board is as follows and it is player 1's turn, player 1 loses, since player 1 cannot place down and X on a vacant spot without making a "three-in-a-row".



   Try playing this game with a friend or a family member. Is there a winning strategy for

   (a) 1 board?
   (b) 2 boards?
   (c) 3 boards?
   (d) x boards?

7. If you have 30 minutes to spare, visit the website here: http://www.ncase.me/trust/ and play through the simulation game. This game uses elements of game theory to explore human psychology. In the lesson and problems, we always want to use the strategy that is most beneficial to us, many times at the cost of others. When we play games with other people, each with their own set of strategy, who will ultimately come out on top? This activity will explore these questions.