Grade 7/8 Math Circles

April 6th, 2022

Inequalities and Absolute Values Extension

Introduction

Last week we solved inequalities and absolute values. Today, we are going to look at solving systems of equations.

Systems of Equations

A **system of equations** is a set of two or more equations that you deal with at one time. A system of equations is required when you want to solve for two or more variables. You need as many equations as you have unknown variables. For example, if you have two unknown variables, x and y, you need to have two equations in order to solve both variables. There are two main ways to solve a system of equations algebraically: by elimination or by substitution.

Solving by Elimination

When we solve by elimination, our goal is to use operations to make the coefficients of one of the variables match or be positive/negative equivalents in order to add or subtract the equations and be left with one variable that we can solve for. Let's do a few examples together to see how this works.

Example 1

Solve for x and y in the following system of equations: $\begin{cases} 2x + 4y = 8 \\ 2x + 2y = 2 \end{cases}$

We see that in both equations, the coefficients of x already match. Therefore, if we subtract the two equations from each other, we are left with an equation that just has y, as follows:

$$\begin{array}{rrcl} & 2x + 4y & = & 8 \\ - & 2x + 2y & = & 2 \\ \hline & 2y & = & 6 \end{array}$$

Now, we have the equation $2y = 6$, which we can divide by 2 on both sides to get that $y = 3$. So, now we have a value for y but we're still missing x!

We can easily solve for x by substituting $y = 3$ back into one of the original equations. So, if we pick the first equation, we get $2x + 4(3) = 8$. Solving for x, we get that $x = -2$.

Now, to check our answer, we can substitute $x = -2$ and $y = 3$ into both equations to make sure they equal the correct thing.

$$2(-2) + 4(3) = -4 + 12 = 8 \qquad\qquad 2(-2) + 2(3) = -4 + 6 = 2$$

These are both exactly what they should be, so we know our final answer is $x = -2$ and $y = 3$.

Example 2

Solve for x and y in the following system of equations: $\begin{cases} 3x + 6y = 8 \\ x - 6y = 4 \end{cases}$

We see that in both equations, the coefficients of y are positive/negative equivalents of each other. Therefore, if we add the two equations from each other, we are left with an equation that just has x, as follows:

$$\begin{array}{rrcl} & 3x + 6y & = & 8 \\ + & x - 6y & = & 4 \\ \hline & 4x & = & 12 \end{array}$$

Now, we have the equation $4x = 12$, which we can divide by 4 on both sides to get that $x = 3$. So, now we have a value for x but we're still missing y!

We can easily solve for y by substituting $x = 3$ back into one of the original equations. So, if we pick the first equation, we get $3(3) + 6y = 8$. Solving for y, we get that $y = -\frac{1}{6}$.

Now, to check our answer, we can substitute $x = 3$ and $y = -\frac{1}{6}$ into both equations to make sure they equal the correct thing.

$$3(3) + 6(-\tfrac{1}{6}) = 9 - 1 = 8 \qquad\qquad (3) - 6(-\tfrac{1}{6}) = 3 + 1 = 4$$

These are both exactly what they should be, so we know our final answer is $x = 3$ and $y = -\frac{1}{6}$.

Example 3

Solve for x and y in the following system of equations: $\begin{cases} 3x + 2y = 10 \\ 2x - y = 2 \end{cases}$

We see that none of the coefficients of the same variable match or are positive/negative equivalents. Therefore, we can multiply one of the equations by a number to make them match or be positive/negative equivalents. In this case, it seems easiest to multiply the second equation by 2 so that we get a $+2y$ and $-2y$, as follows:

$$\begin{aligned} 3x + 2y &= 10 \\ 2 \times (2x - y &= 2) \end{aligned} \qquad \rightarrow \qquad \begin{aligned} 3x + 2y &= 10 \\ 4x - 2y &= 4 \end{aligned}$$

Now we can add the two equations to get an equation that just has x, as follows:

$$\begin{array}{rrcl} & 3x + 2y & = & 10 \\ + & 4x - 2y & = & 4 \\ \hline & 7x & = & 14 \end{array}$$

Now, we have the equation $7x = 14$, which we can divide by 7 on both sides to get that $x = 2$. So, now we have a value for x but we're still missing y!

We can easily solve for y by substituting $x = 2$ back into one of the original equations. So, if we pick the first equation, we get $3(2) + 2y = 10$. Solving for y, we get that $y = 2$.

Now, to check our answer, we can substitute $x = 2$ and $y = 2$ into both equations to make sure they equal the correct thing.

$$3(2) + 2(2) = 6 + 4 = 10 \qquad\qquad 2(2) - (2) = 4 - 2 = 2$$

These are both exactly what they should be, so we know our final answer is $x = 2$ and $y = 2$.

Solving by Substitution

When we solve by substitution, our goal is to isolate x or y in one of the equations and then substitute whatever they are equal to into the other equation. Let's do an example to see how this works.

Example 4

Solve for x and y in the following system of equations: $\begin{cases} 2x + 3y = 9 \\ x + 4y = 2 \end{cases}$

In our second equation, x have no coefficients so it will be easy to isolate it by just moving the $4y$ to the RHS. By doing this, we get $x = 2 - 4y$.
Now, in the *other* equation, in this case the first equation, we substitute $x = 2 - 4y$ and solve for y, as follows:

$$\begin{aligned} 2x + 3y &= 9 \\ 2(2 - 4y) + 3y &= 9 \\ 4 - 8y + 3y &= 9 \\ -5y &= 5 \\ y &= -1 \end{aligned}$$

We have solved that $y = -1$, and we can substitute this into our equation for x to get what x is equal to. So, we have $x = 2 - 4(-1) = 6$.
Now, to check our answer, we can substitute $x = 6$ and $y = -1$ into both equations to make sure they equal the correct thing.

$$2(6) + 3(-1) = 12 - 3 = 9 \qquad (6) + 4(-1) = 6 - 4 = 2$$

These are both exactly what they should be, so we know our final answer is $x = 6$ and $y = -1$.

Exercise

Solve for x and y in the systems of equations below.

(a) $\begin{cases} 2x + 3y = 15 \\ x - 3y = 3 \end{cases}$

(b) $\begin{cases} 2x + y = 5 \\ -3x + 6y = 0 \end{cases}$

Exercise Solution

(a) Using any method, you should get that $x = 6$ and $y = 1$.

(b) Using any method, you should get that $x = 2$ and $y = 1$.