

# Grade 9/10/11 Courseware Lessons

This is a complete list of all the CEMC Grade 9/10/11 mathematics courseware lessons together with the goals of each lesson. There are a total of 144 lessons divided into 29 units across the 7 strands. The lesson titles are hyperlinked.

## Number Sense and Algebraic Expressions

### Unit 1 - Exponents

#### Lesson 1: An Introduction to Exponents

- Examine the relationship between the exponential representation of length, area, and volume.
- Express simplified exponential form in expanded form.
- Represent algebraic expressions in simplified exponential form.

#### Lesson 2: Multiplying and Dividing Monomials

- Explore the exponent rule for multiplying monomials.
- Explore the exponent rule for dividing monomials.

#### Lesson 3: Power of a Power Exponent Rule

- Explore the power of a power rule for both numeric and algebraic expressions.

#### Lesson 4: Negative Bases and Integer Exponents

- Examine powers with positive and negative integer bases.
- Explore the exponent rule for an exponent of zero.
- Examine powers with a negative integer exponent.

#### Lesson 5: Rational Exponents - Part 1

- Define the principal  $n^{\text{th}}$  root of a number.
- Explore rational exponents of the form  $\frac{1}{n}$ .

#### Lesson 6: Rational Exponents - Part 2

- Simplify and evaluate positive rational exponents of the form  $\frac{a}{n}$ .
- Simplify and evaluate negative rational exponents of the form  $-\frac{a}{n}$ .

#### Lesson 7: Exponent Laws All Together

- Simplify algebraic expressions.
- Evaluate numerical expressions.

### Unit 2 - Manipulating Algebraic Expressions

#### Lesson 1: An Introduction to Polynomials

- Introduce the concept of a variable.
- Examine polynomials and classify an expression as monomial, binomial or trinomial based on the number of terms.
- State the degree of a polynomial.

#### Lesson 2: Adding and Subtracting Polynomials

- Look at an expression and identify like terms.
- Add and subtract polynomials by collecting like terms.

#### Lesson 3: Multiplying a Polynomial by a Monomial

- Multiply a polynomial by a monomial using the distributive property.

#### Lesson 4: Multiplying a Polynomial by a Polynomial

- Apply the distributive property to multiply a polynomial by a polynomial.

#### Lesson 5: Simplifying Polynomials

- Simplify polynomials by adding, subtracting, and multiplying.
- Define the term equivalence.
- Determine if two algebraic expressions are equivalent.

## Unit 3 - Radicals and Rational Functions

### Lesson 1: Introduction to Radicals

- Simplify and order radicals involving integers and rational numbers.
- Use technology to estimate the value of a radical.
- Recognize the difference between exact and approximate values.

### Lesson 2: Operations With Radicals

- Add, subtract, and multiply to simplify radical expressions.
- Simplify radical expressions by rationalizing the denominator.

### Lesson 3: Solving Radical Equations

- Define extraneous roots.
- Solve radical equations algebraically and graphically, identifying restrictions on the domain and any extraneous roots.
- Solve real-world application problems involving radical equations.

### Lesson 4: Introduction to Rational Expressions

- Define rational expressions.
- State restrictions on the variable values in a rational expression.
- Simplify rational expressions.
- Determine equivalence in rational expressions.

### Lesson 5: Multiplying and Dividing Rational Expressions

- Multiply and divide rational expressions.
- Simplify these expressions and state restrictions on the variable values.

### Lesson 6: Adding and Subtracting Rational Expressions

- Determine a common denominator for rational expressions.
- Add and subtract rational expressions and state restrictions on the variable(s).
- Simplify rational expressions involving various operations.

## Unit 4 - Prime Factorization

### Lesson 1: Prime Factorization

- Define prime and composite numbers.
- Identify the factors of a composite number.
- Write a composite number as a product of its prime factors using powers.

### Lesson 2: Using Prime Factorization to Determine the GCF and LCM

- Use prime factorization to determine the Greatest Common Factor (GCF) and the Least Common Multiple (LCM) of two or more integers.
- Solve word problems involving GCF's and LCM's.

## Linear Relations and Analytic Geometry

### Unit 1 - Linear Equations

#### Lesson 1: Solving One- and Two-Step Equations

- Solve one and two step linear equations.
- Show a formal check for a solution.

#### Lesson 2: Solving Multi-Step Linear Equations

- Solve multi-step linear equations.
- Examine the process of cross-multiplication and when it can be used.

#### Lesson 3: Applications of Solving Linear Equations

- Solve application problems with the linear equation given.
- Translate a written description to an algebraic equation.
- Solve application problems without the linear equation given.

#### Lesson 4: Solving Problems With Rate, Ratio, Proportion, and Percent

- Review the terms rate, ratio, proportion and percent.
- Solve problems involving rate, ratio, proportion, and percent using cross-multiplication.
- Solve application problems involving rate, ratio, proportion, and percent.

#### Lesson 5: Rearranging Equations and Formulas

- Rearrange linear relations to solve in terms of one variable.
- Rearrange formulas to solve in terms of one variable.

#### Lesson 6: Solving Linear Inequalities

- Solve linear inequalities.
- Solve application problems with and without the linear inequality given.

### Unit 2 - Characteristics of Linear Relations

#### Lesson 1: Introduction to Linear Relations - Part 1

- Use patterns to identify growth rates and starting values in linear relations.
- Develop equations to represent linear patterns or scenarios.
- Create tables of values and corresponding scatter plots for linear relations and look at characteristics within each representation. Revisit how to identify an independent and dependent variable.
- Compare linear and non-linear growth rates.

#### Lesson 2: Introduction to Linear Relations - Part 2

- Define the terms initial value and rate of change.
- Given a linear relation, identify or calculate the initial value and rate of change from scenario descriptions, table of values and graphs, then use the values to develop a corresponding equation.
- Calculate first differences from a table of values and use this concept to determine if a relation is linear or non-linear.

#### Lesson 3: Linear Relations - Direct and Partial Variation

- Define the terms direct variation and partial variation.
- Classify various representations of a linear relation as being a direct or partial variation.

#### Lesson 4: Slope and the y-Intercept

- Define the terms  $y$ -intercept and slope.
- Identify or calculate the  $y$ -intercept and slope of a linear relation given a graph, table of values, or an equation.
- Explore linear families.

#### Lesson 5: Graphing Linear Relations

- Graph linear relations by hand using a table of values.
- Graph linear relations by hand using the  $x$  and  $y$ -intercepts.
- Graph linear relations by hand using the slope and  $y$ -intercept.

### Unit 3 - Connecting Various Representations of Linear Relations

#### Lesson 1: Finding Missing Values in a Linear Relation

- Solve for unknown values in a table of values representing a linear relation.
- Determine information from a linear graph.
- Solve for unknown quantities given a description of a linear relation.

#### Lesson 2: Connecting Various Forms of a Linear Relation

- Identify equivalent representations of a linear relation.
- Connect the table, graph, and equation of a linear relation using the slope and  $y$ -intercept.

#### Lesson 3: Changing the Properties of a Linear Relation

- Determine how changing the slope and/or  $y$ -intercept of a linear relation affects the graph and equation of the relation.

### Unit 4- Properties of Slope

#### Lesson 1: The Slope Formula

- Develop the slope formula for a linear relation.
- Use the slope formula to answer questions about a given linear relation.

## Lesson 2: Working With $y = mx + b$

- Determine algebraically the equation of a line in the form  $y = mx + b$ .

## Lesson 3: Parallel and Perpendicular Lines

- Investigate the properties of slope for both parallel and perpendicular lines.
- Using the properties of slope, solve problems involving parallel and perpendicular lines.

## Lesson 4: Horizontal and Vertical Lines

- Investigate the properties of slope for both horizontal and vertical lines.
- Using the properties of slope, solve problems involving horizontal and vertical lines.

# Unit 5 - Equations of Linear Relations and Problem Solving

## Lesson 1: Alternate Forms of an Equation of a Line

- Identify various forms of an equation of a line.
- Rearrange a given equation of a line from one form to another.
- Solve problems involving the various forms of an equation of a line.

## Lesson 2: Comparing Linear and Non-Linear Relations

- Identify characteristics of a linear relation that distinguish it from a non-linear relation.
- Use tables of values, graphs, or equations to classify a relation as linear or non-linear.

## Lesson 3: Applications of Linear Relations

- Solve problems involving linear relations represented in different forms.
- Determine a point of intersection graphically, and explain the meaning within a given context.
- Identify and explain restrictions on variables within a given context.

## Lesson 4: Interpreting Stories and Graphs

- Given a detailed description of an event, create a corresponding story graph and understand the importance of labeling each axes.
- Given a story graph, create a detailed description of the event.
- Given a story graph that represents a distance vs time scenario, use slope calculations to determine average speed of the objects or people in the question.
- Understand when a graphical representation cannot represent a distance vs time scenario.

# Unit 6 - Solving Linear Systems

## Lesson 1: Solving Linear Systems of Equations Graphically

- Define systems of equations and understand what it means to “solve” one.
- Identify the various ways a system of two linear equations can intersect.
- Solve linear systems of equations involving two variables graphically.

## Lesson 2: Solving Systems of Equations Algebraically

- Solve systems of two equations involving two variables using substitution.
- Solve systems of two equations involving two variables using elimination.
- Understand the result when algebraic methods are used to solve linear systems with no solutions or systems with infinite solutions.

## Lesson 3: Applications of Linear Systems

- Given a description in words, create a linear system of equations to model the scenario.
- Solve a linear system to answer a problem using a variety of methods and interpret the meaning of the solution within the given context.

# Unit 7 - Properties of Line Segments and Using Analytic Geometry to Verify Geometric Properties

## Lesson 1: Determining the Midpoint and Length of a Line Segment

- Define the midpoint of a line segment.
- Develop and use the formula to calculate the midpoint of a line segment.
- Develop and use the formula to calculate the length of a line segment.

## Lesson 2: Problem Solving With Slopes, Lengths, and Midpoints

- Solve problems using the concepts of slope, length, and midpoint of a line segment.

## Lesson 3: Investigating and Verifying Properties of Quadrilaterals

- Determine some characteristics and properties of quadrilaterals.
- Use analytic geometry and algebraic techniques to verify geometric properties of quadrilaterals.

## Lesson 4: Equation of a Circle

- Develop the equation for a circle with  $(0,0)$  and radius  $r$ .
- Given the equation of a circle, identify the radius and draw a sketch.
- Develop the equation for a circle with centre  $(h, k)$  and radius  $r$ .
- Solve problems involving the equation of a circle.

# Unit 8 - Data Management and Statistics

## Lesson 1: Scatter Plots and Lines or Curves of Best Fit

- Construct scatter plots and interpret the meaning of points on scatter plots.
- Identify trends in data and describe the correlation.
- Draw curves or lines of best fit and determine the equation of a line of best fit.
- Interpolate and extrapolate information using the line of best fit.

## Lesson 2: Investigating Relationships Between Two Variables

- Pose problems, identify variables associated with problems, and formulate hypotheses about possible relationships between variables.
- Understand how to design and carry out investigations to collect and organize data to determine if a relationship exists between two variables.

## Lesson 3: Collecting Data, Sampling Bias, and Techniques

- Investigate different types of data.
- Look at bias in data collection.
- Examine different sampling techniques.

## Lesson 4: Display of Data and Representation Bias

- Classify types of data as numerical or categorical.
- Examine various ways to organize and display data.
- Interpret graphs.
- Identify potential problems with the display of data related to bias.

## Lesson 5: Probability in Society

- Explore where probability is used in society.

# Measurement, Geometry, and Trigonometry

## Unit 1 - Pythagorean Theorem, Measurement, and Optimization

### Lesson 1: The Pythagorean Theorem

- Recognize the connections between the geometric and algebraic representations of the Pythagorean Theorem.
- Solve for the missing length of a right triangle.
- Develop and apply the converse of the Pythagorean Theorem.

### Lesson 2: Perimeter and Area of Composite Shapes

- Decompose shapes into simpler shapes with known area formulas.
- Determine the perimeter and area of composite shapes.
- Solve word problems involving perimeters, areas, and unit conversions.

### Lesson 3: Surface Area of Pyramids and Cones

- Visualize the surface area of a pyramid or a cone.
- Calculate the surface areas of pyramids and cones.
- Solve problems involving the surface area of pyramids and cones.

#### Lesson 4: Volume of Pyramids and Cones

- Connect the volume of pyramids and cones to their corresponding prisms and cylinders.
- Calculate the volume of pyramids and cones.
- Solve application problems involving pyramids, cones, and unit conversions.

#### Lesson 5: Volume and Surface Area of Spheres

- Calculate the volume and surface area of spheres.
- Solve word problems involving applications of spheres.

#### Lesson 6: Maximizing Area of Rectangles With Fixed Perimeter

- Recognize that rectangles with the same perimeter may have different areas.
- Represent the relationship between the dimensions of a rectangle with a fixed perimeter and its area using tables of values and graphs.
- Calculate the maximum area of rectangles with a fixed perimeter.

#### Lesson 7: Determining the Optimal Perimeter of Rectangles

- Recognize that rectangles with the same area may have different perimeters.
- Represent the relationship between the dimensions of a rectangle with a fixed area and its perimeter using tables of values and graphs.
- Use systematic trial and formulas to determine the optimal perimeter of rectangles with a fixed area.

#### Lesson 8: Optimizing Surface Area of Cylinders and Square-Based Prisms

- Identify the effect of varying the dimensions on the surface area of a square-based prism or a cylinder with a given volume.
- For a square-based prism or a cylinder with a fixed volume, represent the relationship between the dimensions and surface area using tables of values, graphs, and formulas.
- Calculate the minimum surface area of square-based prisms and cylinders with a fixed volume.

#### Lesson 9: Maximizing the Volume of Cylinders and Square-Based Prisms

- Given the surface area of a cylinder or square-based prism, recognize the optimal shape to maximize the volume.
- For a prism or cylinder with a fixed surface area, represent the relationship between the dimensions and the volume using a table of values and formulas.
- Calculate the maximum volume of prisms and cylinders with a given surface area.

### Unit 2 - Geometric Relationships

#### Lesson 1: Review of Basic Angle Properties

- Recognize the properties of opposite, supplementary and complementary angles.
- Recognize the properties of angles produced by parallel lines and a transversal (i.e, alternate, corresponding and co-interior angles).
- Solve for unknown angles in a diagram using these angle properties.

#### Lesson 2: Angle Properties of Triangles

- Determine the sum of the interior angles of a triangle.
- Determine the sum of the exterior angles of a triangle.
- Explore other relationships between interior and exterior angles, particularly in different types of triangles.
- Use angle relationships in triangles to solve for missing angles, classify triangles, and identify whether conjectures are true or false.

#### Lesson 3: Angle Properties of Quadrilaterals and Other Polygons

- Determine the sum of the interior angles and the sum of the exterior angles of a quadrilateral.
- Identify angle properties in specific types of quadrilaterals (such as parallelograms, squares, etc.).
- Determine the relationship between the number of sides of a polygon and the sum of the interior angles and the sum of the exterior angles.
- Apply angle relationships in the context of regular polygons.
- Use angle relationships in quadrilaterals and other polygons to solve for missing angles and verify whether conjectures are true or false.

#### Lesson 4: Midpoints and Diagonals of Triangles, Quadrilaterals, and Other Polygons

- Investigate and describe the line segments formed by joining the midpoints of the sides of a triangle.
- Investigate and describe the polygon formed by joining the midpoints of the adjacent sides of a quadrilateral.
- Investigate and describe the properties of the diagonals of specific types of quadrilaterals.
- Investigate and describe the number of diagonals that can be drawn in a polygon, depending on the number of sides.
- Use these properties to solve problems and to verify whether conjectures are true or false.

#### Lesson 5: Chords of Circles

- Solve problems involving perpendicular bisectors.
- Determine the relationship between chords and perpendicular bisectors.
- Solve problems involving chords and perpendicular bisectors.

#### Lesson 6: Inscribed and Central Angles of Circles

- Calculate the area of the sector of a circle.
- Calculate arc length within a circle.
- Determine properties of circles related to angles within a circle.
- Solve problems involving angles in circles.

#### Lesson 7: Tangents to Circles

- Determine the relationship between a tangent to a circle and the radius drawn to the point of tangency.
- Solve problems involving tangents of circles.

### Unit 3 - Trigonometry

#### Lesson 1: Similarity and Congruence

- Define congruence and similarity.
- Calculate the scale factor relating two similar polygons.
- Determine the perimeter and area of a polygon using similarity.

#### Lesson 2: Similar Triangles

- Demonstrate that two triangles are similar using similarity rules.
- Use similarity between triangles to solve for an unknown side length.
- Construct an appropriate pair of similar triangles to solve a real-world problem.

#### Lesson 3: Tangent Ratio

- Compute the tangent ratio for an acute angle in a right-angled triangle given the side lengths.
- Use the tangent ratio to solve for an unknown side length in a right-angled triangle.
- Use the inverse tangent operation on your calculator to solve for an interior angle in a right-angled triangle.

#### Lesson 4: Sine and Cosine Ratios

- Compute the sine and cosine ratio for an acute angle in a right-angled triangle given the side lengths.
- Solve for an unknown side length in a right-angled triangle using the sine or cosine ratio.
- Solve for an interior angle in a right-angled triangle using the inverse sine and cosine operations on your calculator.

#### Lesson 5: The Sine Law

- State and prove the sine law.
- Use the sine law to compute one unknown side length or angle in an acute triangle.
- Use the sine law to determine all side lengths and angles in an acute triangle.

#### Lesson 6: The Cosine Law

- State and prove the cosine law.
- Use the cosine law to compute an unknown side length in an acute triangle.
- Use the cosine law to determine the interior angles of an acute triangle.

#### Lesson 7: Applications With Acute Triangles

- Identify when to apply the sine and cosine laws given incomplete information about the side lengths and angles in an acute triangle.
- Solve a multistep problem that involves two or more applications of the sine or cosine laws, interior and exterior angle properties of triangles, or two or more acute triangles.

## Lesson 8: Oblique Triangles

- Compute the sine, cosine, and tangent ratios for obtuse angles.
- Determine the oblique angle or angles corresponding to a given trigonometric ratio.
- Solve an oblique triangle using the sine and cosine laws and correctly handle the ambiguous case of the sine law when encountered.

## Lesson 9: Applications in Three-Dimensional Settings

- Define a set of adjoining triangles to relate unknown lengths and angles to known lengths and angles in a three-dimensional setting.
- Determine a specific unknown length or angle in a three-dimensional setting by applying trigonometric tools to a set of adjoining triangles.

## Unit 4 - Angles in Standard Position and Trigonometric Identities

### Lesson 1: Trigonometric Ratios of Angles in Standard Position

- Draw angles in standard position on the Cartesian plane.
- Determine the primary trigonometric ratios of angles from  $0^\circ$  to  $360^\circ$ .

### Lesson 2: Related and Coterminal Angles

- Define and calculate related acute angles and trigonometric ratios for angles between  $0^\circ$  to  $360^\circ$ .
- Calculate the measure of angles between  $0^\circ$  to  $360^\circ$  from a given trigonometric ratio.
- Define coterminal angles to connect negative angles and angles greater than  $360^\circ$  with angles between  $0^\circ$  to  $360^\circ$ .

### Lesson 3: Trigonometric Ratios of Special Angles

- Recognize connections between the angles and the side lengths of a right isosceles triangle and between the angles and side lengths of an equilateral triangle.
- Draw and find points on the terminal arm of angles in standard position with related acute angles of  $30^\circ$ ,  $45^\circ$ , and  $60^\circ$ .
- Calculate exact values of the sine, cosine, and tangent ratios for angles related to  $30^\circ$ ,  $45^\circ$ ,  $60^\circ$ , and  $90^\circ$ .
- Relate the points on the unit circle to the primary trigonometric ratios of angles in standard position.

### Lesson 4: Reciprocal Trigonometric Ratios

- Define and determine the reciprocal trigonometric ratios for acute angles in right triangles and for angles in standard position.
- Determine the measure of an angle given the value of a reciprocal trigonometric ratio.

### Lesson 5: Trigonometric Identities

- Develop and identify the Pythagorean, quotient, and reciprocal identities.
- Identify and apply strategies to prove trigonometric identities.

## Quadratic Relations

### Unit 1 - Basic Properties of Quadratic Relations

#### Lesson 1: Recognizing Quadratic Relations From Tables of Values

- Create a table of values for a real-life situation represented by a quadratic relation.
- Explore how quadratic relations differ from linear relations and other non-linear relations by examining a graph and by examining a table of values (first and second differences).
- Classify a relation as linear, quadratic, or neither when given its table of values.

#### Lesson 2: Exploring Second Differences

- Collect data that can be represented as a quadratic relation.
- Use first and second differences to determine unknown values in a table.
- Determine if an equation describes a linear or quadratic relation.
- Determine an equation by inspection given a table of values.



### Lesson 3: Properties of Parabolas

- Recognize parabola terminology: vertex, zeros, axis of symmetry, and optimum (maximum or minimum) value.
- Identify the vertex, zeros/ $x$ -intercepts, axis of symmetry and optimum value of a parabola given its table of values or graph.
- Interpret the meanings of a parabola's vertex, zeros/ $x$ -intercepts, axis of symmetry and optimum value within a given context.

### Lesson 4: Comparing $y = x^2$ and $y = 2^x$

- Generate tables of values and graphs for the relations  $y = x^2$  and  $y = 2^x$ .
- Compare features of the graphs of  $y = x^2$  and  $y = 2^x$ .

## Unit 2 - Algebraic Representations of Quadratic Relations

### Lesson 1: Introduction to Standard, Factored, and Vertex Forms

- Verify that a quadratic relation has constant second differences, when given its equation in standard form (i.e., and equation of the form  $y = ax^2 + bx + c$ ).
- Identify the direction of opening of the parabola and the  $y$ -intercept from an equation in standard form.
- Solve problems involving the standard form of a quadratic relation.
- Investigate equivalent equations (e.g., factored form, vertex form) for quadratic relations given in standard form.

### Lesson 2: Exploring Factored Form

- Determine the  $x$ -intercepts/zeros of a quadratic relation given the factored form equation.
- Determine the vertex of a quadratic relation given the factored form equation.
- Determine the factored form equation of a quadratic relation given the  $x$ -intercepts/zeros.

### Lesson 3: Exploring Vertex Form

- Determine the vertex of a quadratic relation given the vertex form equation.
- Determine the vertex form equation of a quadratic relation given the vertex.
- Convert the factored form equation of a quadratic relation to the vertex form equation.

## Unit 3 - Algebraic Skills

### Lesson 1: Expanding and Simplifying

- Review the distributive property in the context of quadratic relations.
- Expand an expression by multiplying or squaring binomials.
- Expand and simplify equations of quadratic relations so that they are in standard form.
- Extend the distributive property beyond multiplying two binomials.

### Lesson 2: Factoring - Common and Trinomials

- Factor an expression using common factoring.
- Factor a trinomial of the form  $x^2 + bx + c$ .
- Factor a trinomial of the form  $ax^2 + bx + c$  with  $a \neq 1$  by decomposition or by inspection.

### Lesson 3: Factoring - Difference of Squares and Perfect Squares

- Factor difference of squares.
- Factor perfect squares.
- Determine which type of factoring applies to a given expression.
- Factor expressions requiring more than one type of factoring.

### Lesson 4: Completing the Square

- Select an appropriate constant to create a trinomial that is a perfect square.
- Write the equation of a quadratic relation in vertex form by completing the square.
- Apply the process of completing the square to answer questions involving vertex, maximum or minimum, or a quadratic relation.

## Unit 4 - Graphing Quadratic Relations

### Lesson 1: Transformations of $y = x^2$

- Determine the image of a set of points under a translation, reflection, and stretch (or compression).
- Determine the role of  $a$  in  $y = ax^2$ .
- Determine the role of  $k$  in  $y = x^2 + k$ .
- Determine the role of  $h$  in  $y = (x - h)^2$ .

## Lesson 2: Graphing and Equations in Vertex Form

- Describe the transformations that are applied to  $y = x^2$  to obtain the graph of  $y = a(x - h)^2 + k$ .
- Sketch the graph of a quadratic relation whose equation is given in the form  $y = a(x - h)^2 + k$ .
- Identify the equation of a quadratic relation when given its graph.

## Lesson 3: Graphing and Equations in Factored Form

- Graph a quadratic relation given in factored form when the zeros are integers.
- Graph a quadratic relation given in factored form when the zeros are not integers.

## Lesson 4: Graphing and Equations in Standard Form

- Sketch the graph of a quadratic relation whose equation is given in standard form,  $y = ax^2 + bx + c$ , by either: Writing the equation in vertex form,  $y = a(x - h)^2 + k$  first, or factoring the equation first.
- Select an appropriate strategy for sketching the graph of a quadratic relation whose equation is given in standard form.

# Unit 5 - Solving Problems Involving Quadratic Relations

## Lesson 1: Solving Quadratic Equations

- Recognize quadratic equations.
- Solve quadratic equations in various forms by graphing, by applying inverse operations, and by factoring.
- Check solutions to quadratic equations by graphing or by performing a formal check.
- Solve application problems that involve solving a quadratic equation.

## Lesson 2: Introduction to the Quadratic Formula

- Derive the quadratic formula.
- Determine the roots of a quadratic equation using the quadratic formula.
- Determine the zeros of a quadratic relation using the quadratic formula.
- Apply the quadratic formula in a variety of contexts.

## Lesson 3: The Number of Zeros of a Quadratic Relation

- Determine the number of zeros of a quadratic relation given its equation written in factored or vertex form.
- Calculate the discriminant of a quadratic relation given in standard form and use it to determine the number of zeros of the relation.
- Given a family of parabolas, determine which members of the family have 0, 1 or 2 zeros.

## Lesson 4: Intersections of Linear and Quadratic Relations

- Identify the possible number of points of intersection between a linear relation and a quadratic relation.
- Identify the point(s) of intersection between a linear relation and quadratic relation both graphically and algebraically.
- Use the discriminant to determine the number of point(s) of intersection between a linear relation and a quadratic relation.

## Lesson 5: Applications

- Use partial factoring to determine the vertex of a quadratic relation.
- Solve problems involving substitution into a quadratic relation.
- Solve problems that require solving a quadratic equation.
- Solve problems that involve finding the maximum or minimum of a quadratic relation.
- Select an appropriate computational strategy depending on the problem.

# Introduction to Functions

## Unit 1 - Representing Functions

### Lesson 1: Introduction to Functions

- Represent relations in a variety of ways, including mapping diagrams; equations; sets of ordered pairs; and graphs.
- Represent relations whose graphs are circles, by using equations, tables and graphs.
- Identify when a relation is a function, by using the definition of a function or the Vertical Line Test.

### Lesson 2: Function Notation

- Describe functions using function notation.
- Analyze linear functions using function notation.
- Analyze quadratic functions using function notation.

### Lesson 3: Domain and Range

- Determine the domain and range of a function containing only a few points.
- Use set notation to describe the domain and range of a given function.
- Determine the domain and range of quadratic functions.

### Lesson 4: Domain and Range of Two New Functions

- Determine the domain and range of square root functions.
- Determine the domain and range of rational functions.

## Unit 2 - Transforming and Graphing Functions

### Lesson 1: Graphing Three Common Functions

- Sketch the graphs of  $f(x) = x^2$ ,  $f(x) = \sqrt{x}$ , and  $f(x) = \frac{1}{x}$ .
- Introduce the idea of an asymptote on a graph.
- Identify the domain and range of the functions  $f(x) = x^2$ ,  $f(x) = \sqrt{x}$ , and  $f(x) = \frac{1}{x}$  using their graphs.

### Lesson 2: Functions and Translations

- Define horizontal and vertical translations, and explore the effects of these transformations on graphs.
- Observe the effect of horizontal and vertical translations on the domain and range of a function.
- Express horizontal and vertical translations in function notation.
- Sketch the graph of a function by applying horizontal and vertical translations to a base graph.

### Lesson 3: Horizontal Stretches, Compressions, and Reflections

- Describe how a reflection in the  $y$ -axis affects a function, and express this type of transformation in function notation.
- Describe how a horizontal stretch or compression affects a function, and express this type of transformation in function notation.
- Sketch graphs by applying a reflection in the  $y$ -axis, and/or horizontal stretch or compression to a known graph of a function.
- Identify the domain and range of a function, after a horizontal stretch or compression and/or reflection in the  $y$ -axis.

### Lesson 4: Vertical Stretches, Compressions, and Reflections

- Describe how a reflection in the  $x$ -axis affects a function, and express this type of reflection in function notation.
- Describe how a vertical stretch or compression affects a function, and express this type of transformation in function notation.
- Sketch graphs by applying a reflection in the  $x$ -axis and/or a vertical stretch or compression to a known graph of a function.
- Identify the domain and range of a function after a vertical stretch or compression and/or reflection in the  $x$ -axis.
- Compare reflections in the  $x$ -axis with reflections in the  $y$ -axis, and compare vertical stretches/compressions to horizontal stretches/compressions.

### Lesson 5: Combining Transformations

- Identify the transformations that are applied to the graph of  $y = f(x)$  to obtain the graph of  $y = af(b(x - h)) + k$ .
- Sketch the graph of a function by applying transformations to a base graph in an appropriate order.
- Identify the domain and range of a transformed function.

## Unit 3 - Inverses of Functions

### Lesson 1: Introduction to Inverses

- Determine the inverse of a function given tables or mapping diagrams.
- Determine the relationship between the graph of a function and the graph of its inverse.
- Determine values of the inverse of  $f(x)$  given an algebraic expression for  $f(x)$ .

### Lesson 2: Determining Inverses of Linear Functions Algebraically

- Determine the inverse of a linear function algebraically.
- Determine the domain and range of the inverse of a function.

### Lesson 3: Inverses of Quadratic Functions

- Determine if the inverse of a function is a function.
- Calculate the inverse of a quadratic function algebraically.
- Restrict the domain of a quadratic function so that the inverse is a function.

## Unit 4 - Inequalities, Absolute Values, and Reciprocals

### Lesson 1: Solving Single-Variable Inequalities

- Express a set of real numbers using interval notation.
- Solve linear inequalities, including compound or simultaneous inequalities, using inverse operations.
- Use different strategies to solve quadratic inequalities, such as graphing, case analysis, or sign analysis.
- Solve applications involving linear and quadratic inequalities.

### Lesson 2: Inequalities in Two Variables

- Determine if an ordered pair is a solution to a two-variable inequality.
- Sketch the graph of a linear or quadratic inequality in two variables.
- Solve application problems that involve a linear or quadratic inequality in two variables.

### Lesson 3: Graphing Reciprocal Functions

- Determine the domain, range and asymptotes of  $f(x) = \frac{1}{x}$ .
- Graph the reciprocal of a linear function.
- Graph the reciprocal of a quadratic function.

### Lesson 4: Graphing Absolute Value Functions

- Evaluate the absolute value of a number.
- Graph absolute value functions of the form  $f(x) = a|b(x - h)| + k$  using transformations.
- Graph the absolute value of a linear function.
- Graph the absolute value of a quadratic function.

### Lesson 5: Solving Absolute Value Equations

- Solve an absolute value equation graphically.
- Solve an equation involving the absolute value of a linear function algebraically.
- Solve an equation involving the absolute value of a quadratic function algebraically.

## Sequences, Series and Financial Literacy

### Unit 1 - Representing Sequences

#### Lesson 1: Introducing Sequences

- Express sequences numerically and graphically, using term notation.
- Represent sequences algebraically, using a recursion formula.
- Represent sequences algebraically, using a general term or function notation.
- Make connections between the different algebraic representations of sequences.

#### Lesson 2: Pascal's Triangle and Binomial Expansions

- Generate Pascal's triangle.
- Identify patterns in Pascal's triangle.
- Expand powers of binomials,  $(a + b)^n$ .
- Determine specific terms in the expansion of  $(a + b)^n$ .

### Unit 2 - Arithmetic and Geometric Sequences and Series and Financial Applications

#### Lesson 1: Arithmetic Sequences

- Identify if a sequence is arithmetic.
- Determine a recursive formula for an arithmetic sequence.
- Determine the general term of an arithmetic sequence.
- Solve questions about arithmetic sequences using the general term.

#### Lesson 2: Banking and Simple Interest

- Describe features of chequing and savings accounts, alternatives to savings accounts, and features of tax-savings investments.
- Connect simple interest, arithmetic sequences and linear growth.
- Solve problems involving simple interest.

### Lesson 3: Geometric Sequences

- Identify a geometric sequence.
- Determine a recursion formula for a geometric sequence.
- Determine and apply the formula for the general term of a geometric sequence.
- Solve problems involving geometric sequences.

### Lesson 4: Compound Interest

- Define compound interest, and compare it to simple interest.
- Develop and use a formula for compound interest.
- Connect compound interest, geometric sequences and exponential growth.
- Calculate the future value, present value or interest rate algebraically, in contexts with varying compounding periods.
- Calculate the number of compounding periods graphically.

### Lesson 5: Arithmetic Series

- Define a series as the sum of the terms of a sequence.
- Derive two formulas for the sum of the first  $n$  terms of an arithmetic series.
- Solve problems using the formulas for the sum of the first  $n$  terms of an arithmetic series.

### Lesson 6: Geometric Series

- Define a geometric series.
- Derive a formula for the sum of the first  $n$  terms in a geometric series ( $S_n$ ) and use this formula to calculate sums of given geometric series.
- Solve problems involving the application of geometric series.

### Lesson 7: Solving Annuity Problems as Geometric Series

- Identify different types of annuities.
- Design spreadsheets, including amortization tables, to calculate the balance of an annuity.
- Solve for the future value and present value of an annuity using the geometric series formula.
- Derive formulas for the future value and present value of an ordinary simple annuity using the geometric series formula.
- Apply the formulas for the future value and present value of an ordinary simple annuity to solve annuity problems.

### Lesson 8: Solving Annuity Problems With Technology

- Use technology such as a TVM solver to calculate the future value, present value, regular payment, number of payments or interest rate of an annuity.
- Use technology to compare annuities; in particular, the total amount of interest earned or charged under different conditions.
- Solve annuity problems involving multiple calculations.

### Lesson 9: Other Financial Topics

- Define different types of employment income (i.e., salary, hourly wages, commission and piece rates) and make calculations relating to each of these.
- Identify common Canadian deductions (such as income tax, EI, and CPP), calculate the amounts of these deductions based on earnings, and calculate net pay.
- Compare the advantages and disadvantages of buying, renting, or leasing for major expenses such as housing and vehicles.

## Exponential and Trigonometric Functions

### Unit 1 - Exponential Functions

#### Lesson 1: Introduction to Exponential Functions

- Define exponential growth and exponential decay and determine a function describing these processes.
- Evaluate an exponential function with a particular input to determine the outcome of an exponential growth or decay process.

#### Lesson 2: Properties of Basic Exponential Functions

- Determine if an exponential function of the form  $f(x) = a \times c^x$  can be evaluated when  $x$  is a negative integer or a rational number.
- Determine the range of an exponential function of the form  $f(x) = a \times c^x$  with  $a > 0$ .
- Locate any intercepts and asymptotes of an exponential function of the form  $f(x) = a \times c^x$  with  $a > 0$ .

### Lesson 3: Identifying Exponential Functions

- Determine the equation of an exponential function given a table of values or a graph.
- Use finite differences to determine if a data set is representative of an exponential process.

### Lesson 4: Transformations of Exponential Functions

- Determine an equation for an exponential function that has undergone reflection, stretch, and translation transformations.
- Determine the domain and range of a transformed exponential function.
- Graph a transformed exponential curve.

### Lesson 5: Comparing Exponential Functions

- Express a given exponential function using a different base.
- Find an exponential function with a base greater than one which describes exponential decay.

### Lesson 6: Modelling With Exponential Functions

- Given a description of an exponential process, determine an appropriate form for a transformed exponential function which models this process.
- Fix the base and parameters of a transformed exponential function so as to accurately describe an exponential process.

## Unit 2 - Sinusoidal Functions

### Lesson 1: Periodic Functions

- Classify and sketch graphs of periodic functions.
- Identify the properties of periodic functions: cycle, period, axis, and amplitude.
- Use the properties of periodic functions to extrapolate values outside a given domain.

### Lesson 2: The Sine and Cosine Functions

- Connect the coordinates of points on the unit circle to the numerical and graphical representations of the sine function and the cosine function.
- Identify key properties of the sine and cosine functions, such as amplitude, period, and axis.

### Lesson 3: Investigate Transformations of Sinusoidal Functions

- Observe the effects of vertical and horizontal reflections, stretches, and compressions of the graphs of  $f(x) = \sin(x^\circ)$  and  $f(x) = \cos(x^\circ)$ .
- Identify the properties (amplitude, period, equation of axis, and range) of sinusoidal functions from a graph or an equation.
- Graph simple transformations of  $f(x) = \sin(x^\circ)$  and  $f(x) = \cos(x^\circ)$  from a given equation.
- Define phase shift and explore the effects of translations on the graphs of sinusoidal functions.

### Lesson 4: Graphing Sinusoidal Functions

- Determine the amplitude, period, phase shift, equation of axis, and range of sinusoidal functions in the form  $y = a \sin(b(x - h)^\circ) + k$  or  $y = a \cos(b(x - h)^\circ) + k$ .
- Sketch the graph of  $y = af(b(x - h)^\circ) + k$  by applying transformations to the graphs of  $f(x) = \sin(x^\circ)$  and  $f(x) = \cos(x^\circ)$  or applying the properties to identify key points.

### Lesson 5: Modelling Periodic Behaviour

- Determine the equation of a sinusoidal function given its properties or its graph.
- Relate real-world periodic behaviour to the properties of sinusoidal functions.

### Lesson 6: Applications of Sinusoidal Functions

- Relate the properties of sinusoidal functions to the characteristics of real-world situations.
- Determine a sinusoidal function to model data that demonstrates periodic behaviour.