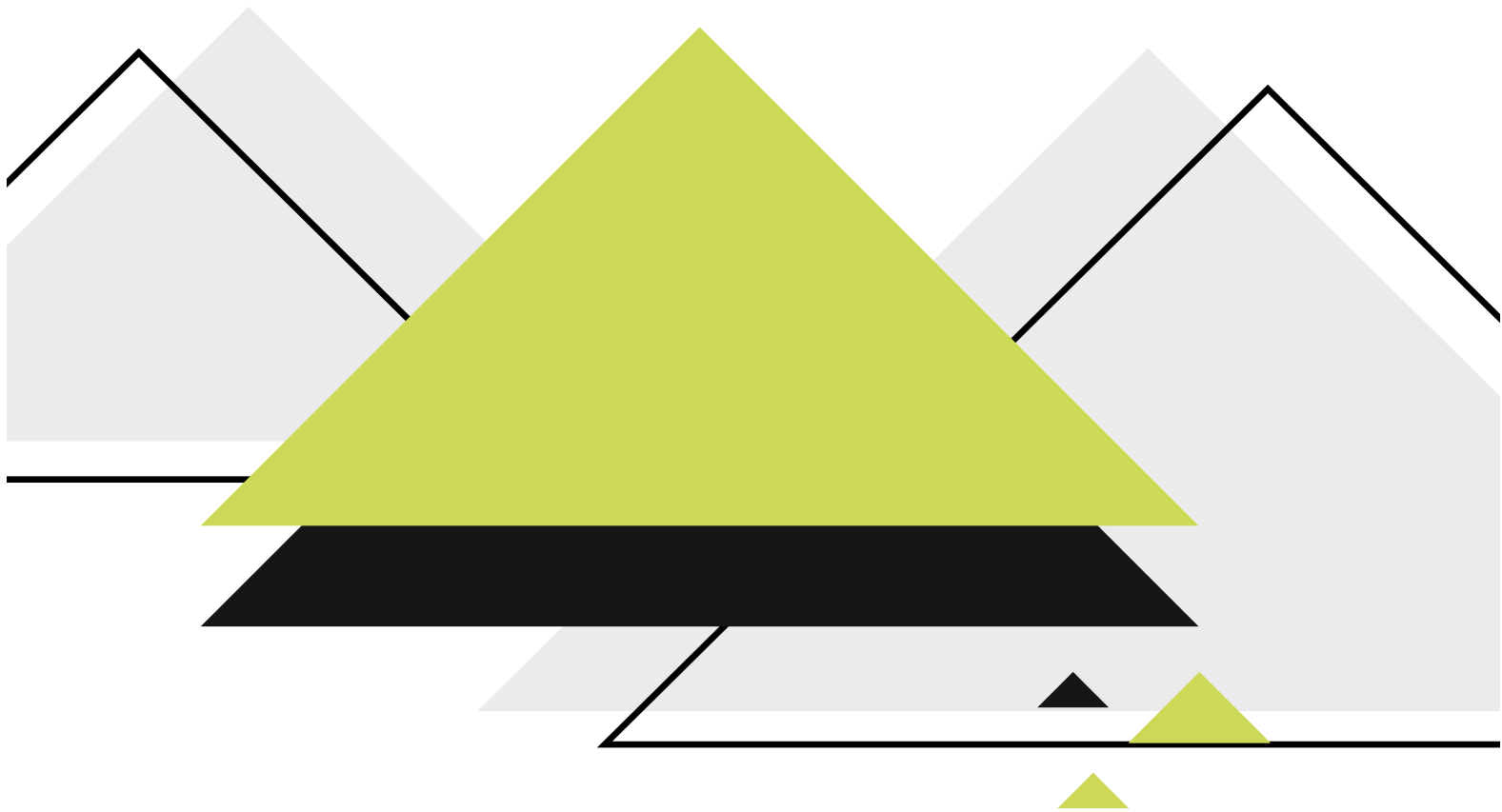


Problem of the Week

Problems and Solutions 2023-2024



Problem A

Grade 3/4



The CENTRE for EDUCATION
in MATHEMATICS and COMPUTING
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Table of Contents

The problems in this booklet are organized into themes.
A problem often appears in multiple themes.
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[Data Management \(D\)](#)

[Geometry & Measurement \(G\)](#)

[Number Sense \(N\)](#)



Algebra (A)

**Take me to the
cover**



Problem of the Week

Problem A

Messy Mosaic

A pattern of figures is made from square blocks. Here are the first four figures of the pattern:

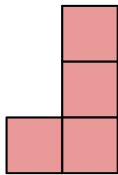


Figure 1

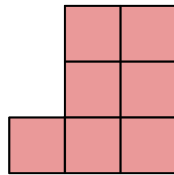


Figure 2

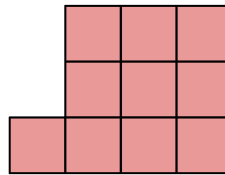


Figure 3

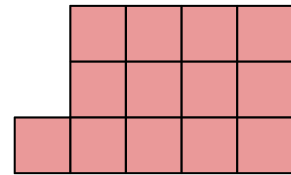
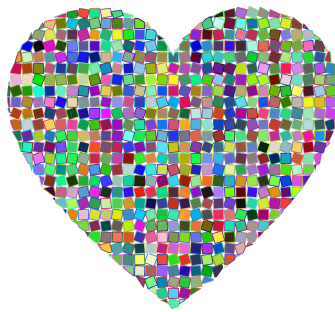
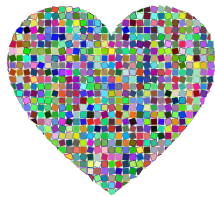


Figure 4

- Describe a *pattern rule* for the pattern.
- Using your pattern rule from part (a), what would Figure 10 be in the pattern?
- Using your pattern rule from part (a), how many blocks are in Figure 100?





Problem of the Week

Problem A and Solution

Messy Mosaic

Problem

A pattern of figures is made from square blocks. Here are the first four figures of the pattern:

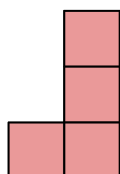


Figure 1

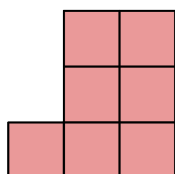


Figure 2

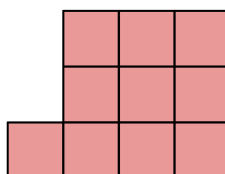


Figure 3

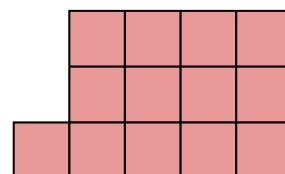


Figure 4

- (a) Describe a *pattern rule* for the pattern.
- (b) Using your pattern rule from part (a), what would Figure 10 be in the pattern?
- (c) Using your pattern rule from part (a), how many blocks are in Figure 100?

Solution

- (a) Here is one possible pattern rule. Each figure has a single column that is one block high, followed by columns that are three blocks high. Each figure of the pattern has one more three-block column than the previous figure.
- (b) Using the pattern we described in part (a), we could draw pictures of the fifth through ninth figures before drawing the tenth figure. However, we notice that Figure 1 has 1 three-block column, Figure 2 has 2 three-block columns, Figure 3 has 3 three-block columns, and Figure 4 has 4 three-block columns. From this, we can extrapolate that Figure 10 has 10 three-block columns.

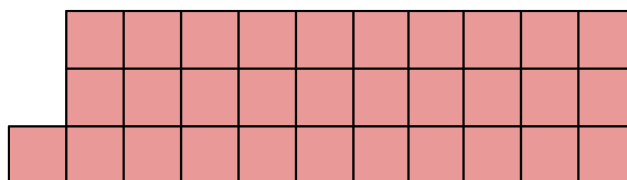


Figure 10

- (c) Using the same logic as in part (b), there will be 100 three-block columns in Figure 100, giving a total of $3 \times 100 = 300$ blocks. When we include the first column in our count, this means there are 301 blocks in Figure 100.



Teacher's Notes

This problem provides a visual example of an *arithmetic sequence*. An arithmetic sequence is a sequence of numbers in which each number after the first is obtained from the previous number by adding a constant, called the common difference. In this problem, the common difference is 3.

The mathematical expression we can use to describe the general form of the n^{th} term in an arithmetic sequence is

$$a_n = a_1 + (n - 1)d$$

where a_1 is the first term of the sequence, d is the common difference between pairs of numbers in the sequence, and a_n is the n^{th} term of the sequence.

In this problem, the first term of the sequence is 4 and the common difference is 3, so the mathematical expression for the n^{th} term of the sequence is

$$a_n = 4 + (n - 1)(3)$$

We can use this formula to calculate the number of blocks in Figure 10:

$$a_{10} = 4 + (10 - 1)(3) = 4 + (9)(3) = 4 + 27 = 31$$

We can also use this formula to calculate the number of blocks in Figure 100:

$$a_{100} = 4 + (100 - 1)(3) = 4 + (99)(3) = 4 + 297 = 301$$



Problem of the Week

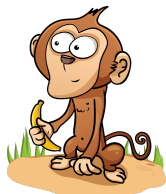
Problem A

Banana Bonanza

Three monkeys found a tree that has 54 bananas growing on it. Each monkey eats two bananas a day from the tree, and no other animals eat any of the bananas.

- (a) How many bananas will be left on the tree at the end of the first day?
- (b) If they start eating from the tree on a Monday, on what day of the week will they eat the last banana?





Problem of the Week

Problem A and Solution

Banana Bonanza

Problem

Three monkeys found a tree that has 54 bananas growing on it. Each monkey eats two bananas a day from the tree, and no other animals eat any of the bananas.

- (a) How many bananas will be left on the tree at the end of the first day?
- (b) If they start eating from the tree on a Monday, on what day of the week will they eat the last banana?

Solution

- (a) Since each monkey eats 2 bananas and there are 3 monkeys, this means $2 + 2 + 2$ or $2 \times 3 = 6$ bananas are eaten from the tree each day. Since the tree started with 54 bananas, this means it will have $54 - 6 = 48$ bananas left at the end of the first day.
- (b) We can make a table to keep track of how many bananas are left on the tree at the end of each day, knowing that the monkeys eat 6 bananas a day. From part (a), we know that there are 48 bananas left at the end of the first Monday.

Day	Bananas Left at End of Day
Monday	48
Tuesday	42
Wednesday	36
Thursday	30
Friday	24
Saturday	18
Sunday	12
Monday	6
Tuesday	0

From this table, we can see that they will eat the last banana on a Tuesday.



Problem of the Week

Problem A

Feeding the Pets

I give my dog 162 g of dog food each meal. He eats two meals per day.

I give my cat 95 g of cat food each meal. She eats one meal per day.

The dog food comes in a 5 kg bag. The cat food comes in a 2 kg bag.

If I start a new bag of food for each pet on the same day, which bag will run out first? Justify your answer.





Problem of the Week

Problem A and Solution

Feeding the Pets

Problem

I give my dog 162 g of dog food each meal. He eats two meals per day.

I give my cat 95 g of cat food each meal. She eats one meal per day.

The dog food comes in a 5 kg bag. The cat food comes in a 2 kg bag.

If I start a new bag of food for each pet on the same day, which bag will run out first? Justify your answer.

Solution

One way to determine which bag will run out first is to make a table that keeps track of how much food each pet consumes.

Since the dog eats two meals of 162 g each day, then he consumes $162 + 162 = 324$ g per day. Since $1 \text{ kg} = 1000 \text{ g}$, then $2 \text{ kg} = 2 \times 1000 = 2000 \text{ g}$ and $5 \text{ kg} = 5 \times 1000 = 5000 \text{ g}$. We will continue tracking the amount of food consumed until we reach one of these limits.

Day	Total Consumed by Cat (in g)	Total Consumed by Dog (in g)
1	95	324
2	190	648
3	285	972
4	380	1296
5	475	1620
6	570	1944
7	665	2268
8	760	2592
9	855	2916
10	950	3240
11	1045	3564
12	1140	3888
13	1235	4212
14	1330	4536
15	1425	4860
16	1520	5184

On Day 16 the dog will have consumed more than 5000 g of food, but the cat has not yet consumed 2000 g of food. So the dog food bag will run out first.

Alternatively, we could use estimation to determine which bag will run out first. Since 95 g is less than 100 g, then the cat food will last at least 20 days since $20 \times 100 = 2000$. Since 324 g is greater than 250 g, then the dog food will last less than 20 days since $20 \times 250 = 5000$.

Therefore, the cat food will last longer than the dog food, so the dog food bag will run out first.



Problem of the Week

Problem A

Bicycle Time

Mr. Turnblatt's bicycle needs to be serviced every 600 km. Each year, he rides his bike between March and November for 35 consecutive weeks, 5 days a week, 8 km a day.

- (a) If he has his bike serviced before his first trip of the year, how many weeks will it be until his bicycle needs the next service?
- (b) How many times will he have his bike serviced in one year?





Problem of the Week

Problem A and Solution

Bicycle Time

Problem

Mr. Turnblatt's bicycle needs to be serviced every 600 km. Each year, he rides his bike between March and November for 35 consecutive weeks, 5 days a week, 8 km a day.

- (a) If he has his bike serviced before his first trip of the year, how many weeks will it be until his bicycle needs the next service?
- (b) How many times will he have his bike serviced in one year?

Solution

- (a) Since Mr. Turnblatt rides his bike 5 days a week and 8 km a day, then in one week he travels $5 \times 8 = 40$ km.

We can skip count by 40 to see how many weeks it takes to get to 600 km:

40, 80, 120, 160, 200, 240, 280, 320, 360, 400, 440, 480, 520, 560, 600

So after 15 weeks Mr. Turnblatt will have ridden 600 km and will need to have his bike serviced.

Alternatively, we could have divided $600 \div 40 = 15$ to determine that it takes Mr. Turnblatt 15 weeks to ride 600 km.

- (b) From part (a), we know that his bicycle needs to be serviced every 15 weeks. So, the bike needs to be serviced at the end of Week 15 and at the end of Week 30. After that, since there are only 5 more weeks of biking, the bike does not need to be serviced until the following year.

Counting the service before the first trip of the year, Mr. Turnblatt will need to have his bike serviced 3 times in a year.



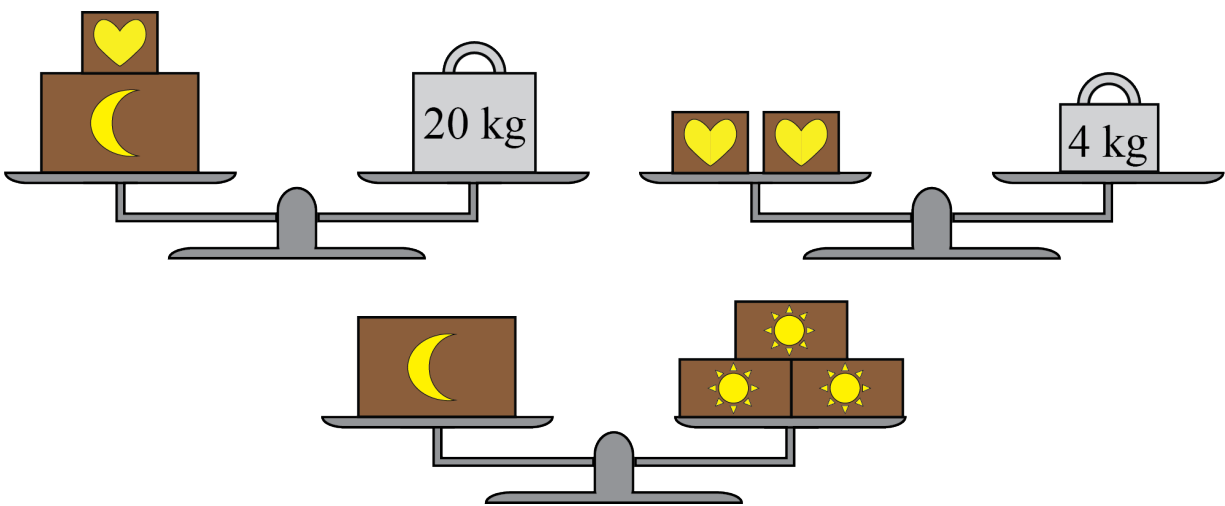
Problem of the Week

Problem A

Balancing Act

Robbie is in charge of sending out boxes from a distribution centre. The contents of the boxes are identified by shapes stamped on them: a heart, a moon, or a sun. All boxes with the same stamp have the same mass.

The following diagrams show what Robbie observed when arranging some of the boxes and standard weights on a scale.



Given that each scale is balanced, determine the mass of each box.



Problem of the Week

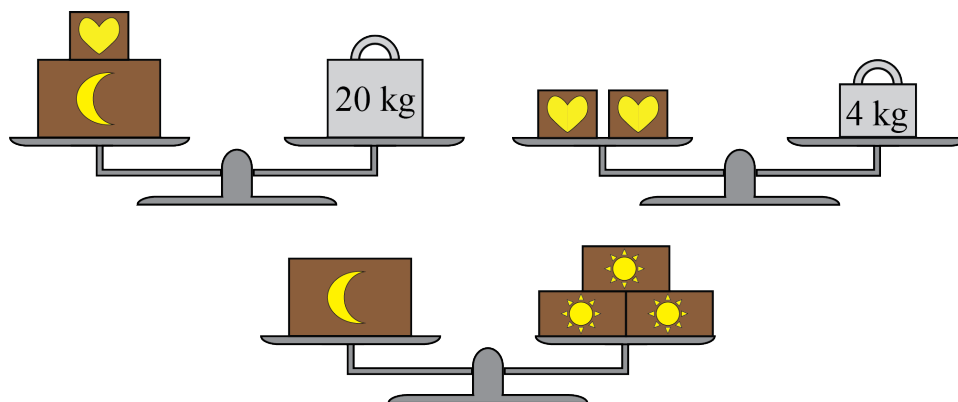
Problem A and Solution

Balancing Act

Problem

Robbie is in charge of sending out boxes from a distribution centre. The contents of the boxes are identified by shapes stamped on them: a heart, a moon, or a sun. All boxes with the same stamp have the same mass.

The following diagrams show what Robbie observed when arranging some of the boxes and standard weights on a scale.



Given that each scale is balanced, determine the mass of each box.

Solution

From the diagrams we notice the following.

- One moon box and one heart box have a total mass of 20 kg.
- Two heart boxes have a mass of 4 kg.
- One moon box has the same total mass as three sun boxes.

Since two heart boxes have a mass of 4 kg, then the mass of one heart box is $\frac{1}{2}$ of 4 kg. Therefore, one heart box has a mass of 2 kg.

Since one heart box and one moon box have a total mass of 20 kg, and one heart box has a mass of 2 kg, then one moon box has a mass of $20 - 2 = 18$ kg.

Since 3 sun boxes have the same total mass as one moon box, then 1 sun box must be $\frac{1}{3}$ the mass of one moon box. Since $\frac{1}{3}$ of 18 is 6, one sun box must have a mass of 6 kg.

Therefore, one heart box has a mass of 2 kg, one moon box has a mass of 18 kg, and one sun box has a mass of 6 kg.



Teacher's Notes

The idea of a balance scale is a nice analogy for an algebraic equation. We can represent the information in the problem using equations with variables to represent the masses of the different types of boxes. Here is one way to solve the problem algebraically.

Let x represent the mass of a heart box, in kg.

Let y represent the mass of a sun box, in kg.

Let z represent the mass of a moon box, in kg.

From the information in the diagrams, we can write the following equations:

$$x + z = 20 \quad (1)$$

$$2x = 4 \quad (2)$$

$$z = 3y \quad (3)$$

We can divide both sides of equation (2) by 2 to get

$$\begin{aligned} \frac{2x}{2} &= \frac{4}{2} \\ x &= 2 \end{aligned}$$

Now, substituting $x = 2$ into equation (1), we get

$$2 + z = 20$$

Subtracting 2 from each side of this equation, we get

$$z = 18$$

Finally, substituting $z = 18$ into equation (3), we get

$$18 = 3y$$

Dividing both sides of this equation by 3, we get

$$\begin{aligned} \frac{18}{3} &= \frac{3y}{3} \\ 6 &= y \end{aligned}$$

Therefore, one heart box has a mass of 2 kg, one moon box has a mass of 18 kg, and one sun box has a mass of 6 kg.



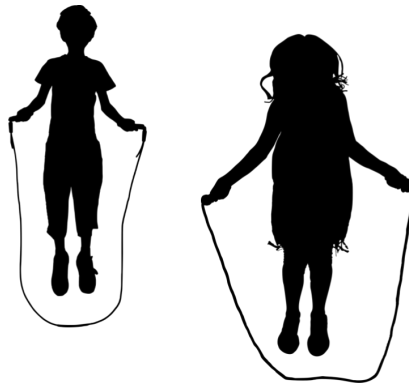
Problem of the Week

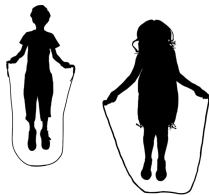
Problem A

Skipping at School

Sophia is an excellent skipper and skips every morning recess with her friends at school. At the end of each recess, she counts how many times she has jumped. Sophia rarely misses her jumps or gets caught on the skipping rope, and so she averages 60 jumps every minute.

- (a) If recess is 10 minutes long, how many jumps do you expect Sophia to complete in a single recess?
- (b) How many days will it take for Sophia to complete at least 5000 jumps, if she jumps for one recess every day?





Problem of the Week

Problem A and Solution

Skipping at School

Problem

Sophia is an excellent skipper and skips every morning recess with her friends at school. At the end of each recess, she counts how many times she has jumped.

Sophia rarely misses her jumps or gets caught on the skipping rope, and so she averages 60 jumps every minute.

- (a) If recess is 10 minutes long, how many jumps do you expect Sophia to complete in a single recess?
- (b) How many days will it take for Sophia to complete at least 5000 jumps, if she jumps for one recess every day?

Solution

- (a) Since Sophia jumps an average of 60 times in 1 minute, and recess is 10 minutes long, we can expect Sophia to complete $60 \times 10 = 600$ jumps in a single recess.
- (b) From part (a) we expect Sofia to complete 600 jumps in one recess. Since she jumps for one recess every day, then in two days, we would expect her to complete $600 + 600 = 1200$ jumps. We can make a table to determine how long it will take Sophia to complete at least 5000 jumps.

Days	Total Jumps
1	600
2	$600 + 600 = 1200$
3	$1200 + 600 = 1800$
4	$1800 + 600 = 2400$
5	$2400 + 600 = 3000$
6	$3000 + 600 = 3600$
7	$3600 + 600 = 4200$
8	$4200 + 600 = 4800$
9	$4800 + 600 = 5400$

After 8 days, we would expect Sophia to have completed 4800 jumps, and after 9 days we would expect Sophia to have completed 5400 jumps. Therefore, it will take Sophia 9 days to complete at least 5000 jumps.



Problem of the Week

Problem A

Babysitting Bonus

Anya agrees to look after her younger brother every day for 1 hour before dinner. Her parents agree to pay her \$15 per week, starting in September. If she does a spectacular job, her parents agree that on the first Monday of each month she will get a raise of \$2 per week. So far, Anya has done a spectacular job.

Anya's parents have a monthly household budget. In the budget, they estimate how much they will be spending on different things each month. Determine the first month when her parents should estimate spending more than \$100 per month for paying Anya to babysit. Justify your answer.





Problem of the Week

Problem A and Solution

Babysitting Bonus

Problem

Anya agrees to look after her younger brother every day for 1 hour before dinner. Her parents agree to pay her \$15 per week, starting in September. If she does a spectacular job, her parents agree that on the first Monday of each month she will get a raise of \$2 per week. So far, Anya has done a spectacular job.

Anya's parents have a monthly household budget. In the budget, they estimate how much they will be spending on different things each month. Determine the first month when her parents should estimate spending more than \$100 per month for paying Anya to babysit. Justify your answer.

Solution

We will estimate how much Anya earns in each month by approximating that there are 4 weeks in one month. The estimated monthly earnings are summarized in the table below.

Month	Earnings per Week	Estimated Earnings per Month
September	\$15	$4 \times \$15 = \60
October	$\$15 + \$2 = \$17$	$4 \times \$17 = \68
November	$\$17 + \$2 = \$19$	$4 \times \$19 = \76
December	$\$19 + \$2 = \$21$	$4 \times \$21 = \84
January	$\$21 + \$2 = \$23$	$4 \times \$23 = \92
February	$\$23 + \$2 = \$25$	$4 \times \$25 = \100
March	$\$25 + \$2 = \$27$	$4 \times \$27 = \108

Using these estimations, Anya's parents should budget more than \$100 per month starting in March.

NOTE: There is a possibility that either December or January will have five Mondays. (During a leap year, there could possibly be five Mondays in February.) In this case, Anya's parents would be paying her more than \$100 for babysitting during that month. However, the budget is an estimation of expenses, so it is reasonable for them to start budgeting more than \$100 for Anya's babysitting starting in March.

The background features a complex arrangement of 3D cubes in various shades of blue and black, creating a sense of depth and perspective. A dark, textured banner with a white border is positioned horizontally across the middle of the image. The text is centered on this banner.

Computational Thinking (C)

A dark, rounded rectangular button with a white arrow pointing upwards, containing the text "Take me to the cover".

Take me to the cover



Problem of the Week

Problem A

What Number Am I?

I am a 4-digit number.

My ones digit plus my thousands digit is equal to my tens digit plus my hundreds digit.

My tens digit is twice as much as my thousands digit.

My ones digit is the largest single-digit whole number.

My thousands digit is 4.

What number am I?





Problem of the Week

Problem A and Solution

What Number Am I?

Problem

I am a 4-digit number.

My ones digit plus my thousands digit is equal to my tens digit plus my hundreds digit.

My tens digit is twice as much as my thousands digit.

My ones digit is the largest single-digit whole number.

My thousands digit is 4.

What number am I?

Solution

We are given that the ones digit is the largest single-digit whole number. Since the largest single-digit whole number is 9, the ones digit must be a 9.

We are also given that the thousands digit is a 4. Since the tens digit is twice as much as the thousands digit, then the tens digit must be $2 \times 4 = 8$.

Also, the sum of the ones digit and the thousands digit is $9 + 4 = 13$. Thus, the sum of the hundreds digit and the tens digit must be 13.

Since the tens digit is 8, then the hundreds digit must be $13 - 8 = 5$.

Thus, the thousands digit is 4, the hundreds digit is 5, the tens digit is 8, and the ones digit is 9. Therefore, the number is 4589.



Problem of the Week

Problem A

Dated Messages

A *Caesar Cipher* is a way to create secret messages by shifting letters in text. For example, a Caesar Cipher of 3 shifts each letter in the text by 3. If you want to shift the letter **D** by 3, then you count three letters forward to arrive at the letter **G**. Similarly, if you want to shift the letter **E** by 3, then you count three letters forward to arrive at the letter **H**. So in a Caesar Cipher of 3, the letter **D** is encoded with the letter **G**, the letter **E** is encoded with the letter **H**, and so on. When shifting letters, if you reach the end of the alphabet, you continue counting at the letter **A**. For example, if you want to shift the letter **Y** by 3, then you count forward to **Z**, then to **A**, and end up at the letter **B**.

- (a) Using a Caesar Cipher of 3, encode the message **FRACTIONS**.
- (b) To decode a secret message you shift the letters in the opposite direction. For example, in a Caesar Cipher of 4 the letter **G** would be decoded as **C**. Decode the message **AEXIVPSS** using a Caesar Cipher of 4.
- (c) A *Date Cipher* shifts the letters in a message by the corresponding digit of a date in the form *YYYYMMDD*. If the message is longer than the date, then we repeat the date as many times as necessary. In the table below, the message **FRACTIONS** has been encoded using the digits from the International Women's Day, 20240308.

Original Letter	F	R	A	C	T	I	O	N	S
Digit of Date	2	0	2	4	0	3	0	8	2
Encoded Letter	H	R	C	G	T	L	O	V	U

The secret message for **FRACTIONS** would be **HRCGTLOVU**.

A famous mathematician has the birthdate December 9, 1906 (19061209).

Use the Date Cipher and this date to **decode** the message

HAAIFJOYQNR to find the name of the famous mathematician.



Problem of the Week

Problem A and Solution

Dated Messages

Problem

A *Caesar Cipher* is a way to create secret messages by shifting letters in text. For example, a Caesar Cipher of 3 shifts each letter in the text by 3. If you want to shift the letter **D** by 3, then you count three letters forward to arrive at the letter **G**. Similarly, if you want to shift the letter **E** by 3, then you count three letters forward to arrive at the letter **H**. So in a Caesar Cipher of 3, the letter **D** is encoded with the letter **G**, the letter **E** is encoded with the letter **H**, and so on. When shifting letters, if you reach the end of the alphabet, you continue counting at the letter **A**. For example, if you want to shift the letter **Y** by 3, then you count forward to **Z**, then to **A**, and end up at the letter **B**.

- (a) Using a Caesar Cipher of 3, encode the message **FRACTIONS**.
- (b) To decode a secret message you shift the letters in the opposite direction. For example, in a Caesar Cipher of 4 the letter **G** would be decoded as **C**.
Decode the message **AEXIVPSS** using a Caesar Cipher of 4.
- (c) A *Date Cipher* shifts the letters in a message by the corresponding digit of a date in the form *YYYYMMDD*. If the message is longer than the date, then we repeat the date as many times as necessary. In the table below, the message **FRACTIONS** has been encoded using the digits from the International Women’s Day, 20240308.

Original Letter	F	R	A	C	T	I	O	N	S
Digit of Date	2	0	2	4	0	3	0	8	2
Encoded Letter	H	R	C	G	T	L	O	V	U

The secret message for **FRACTIONS** would be **HRCGTLOVU**.

A famous mathematician has the birthdate December 9, 1906 (19061209). Use the Date Cipher and this date to **decode** the message **HAAIFJOYQNR** to find the name of the famous mathematician.

Solution

- (a) We encode the message by shifting each letter by 3. The results are summarized in the table below.

Original Letter	F	R	A	C	T	I	O	N	S
Encoded Letter	I	U	D	F	W	L	R	Q	V

So the encoded message is **IUDFWLRQV**.



- (b) We decode the message by shifting each letter by 4 in the opposite direction. The results are summarized in the table below.

Coded Letter	A	E	X	I	V	P	S	S
Decoded Letter	W	A	T	E	R	L	O	O

So the decoded message is **WATERLOO**.

- (c) We decode the message by shifting each letter in the opposite direction by the corresponding digit of the date. The results are summarized in the table below.

Original Letter	H	A	A	I	F	J	O	Y	Q	N	R
Digit of Date	1	9	0	6	1	2	0	9	1	9	0
Decoded Letter	G	R	A	C	E	H	O	P	P	E	R

So the decoded message is **GRACE HOPPER**.

Note that we can check that we have decoded properly by encoding the message to make sure we get the original secret message that was sent to us.



Teacher's Notes

The answer when decoding the secret message in part (c) is Grace Hopper.

Grace Murray Hopper was a pioneer in the early days of Computer Science. She joined the U.S. Navy during World War II, and eventually achieved the rank of Admiral. World War II was a catalyst for the rapid progression towards the modern digital computer. In 1944, Grace Hopper was part of the team who worked on the Harvard Mark I which was one of the earliest general purpose electromechanical computers.





One of Hopper's most notable contributions in the history of Computer Science is her work with compilers. A compiler converts a program that is written in a programming language that is more English-like, and is reasonably easy for humans to read into machine language. Machine language is a sequence of zeros and ones. Before compilers were created, people had to write programs in assembly language, which had instructions like: `addi`, `beq`, or `lw`. Assembly code programs took many more instructions to accomplish the same task as modern programming languages. A compiler made coding more accessible and programs easier to write and modify. Grace Hopper created one of the earliest compilers for a language called A-0.

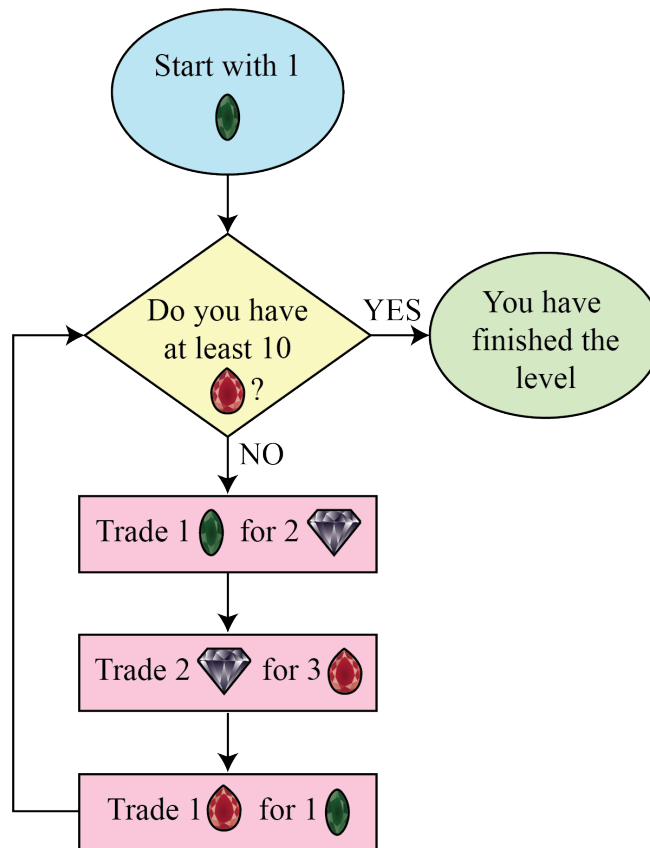


Problem of the Week

Problem A

Gathering Treasure

Genevieve is making a video game where players need to trade gems in order to get to the next level. The gems in the game are emeralds , diamonds , and rubies . In the first level, players make three trades of their gems, as shown in the diagram, until they have at least 10 rubies .



- (a) How many of each gem will a player have when they finish the first level?
- (b) How many trades in total will a player have made when they finish this level?







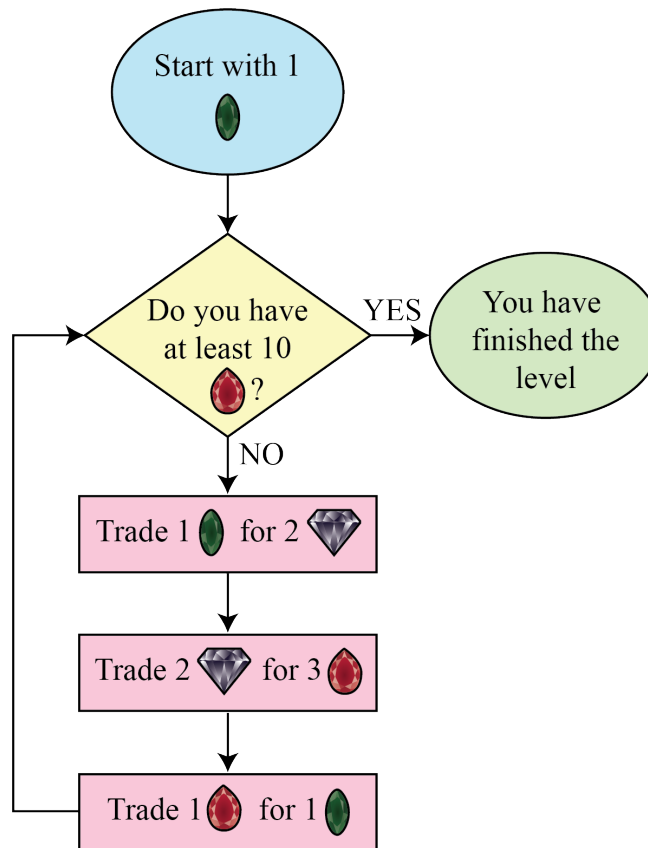
Problem of the Week

Problem A and Solution

Gathering Treasure

Problem



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





Solution




- (a) The player starts with only 1 , so does not have at least 10 , and therefore needs to make trades. The gems after the first three trades are shown.

$$1 \text{ } \rightarrow 2 \text{ } \rightarrow 3 \text{ } \rightarrow 1 \text{ , } 2 \text{ $$

The player still does not have at least 10 , so needs to make trades. The gems after the next three trades are shown.

$$1 \text{ , } 2 \text{ } \rightarrow 2 \text{ , } 2 \text{ } \rightarrow 5 \text{ } \rightarrow 1 \text{ , } 4 \text{ $$

The player still does not have at least 10 , so needs to make trades. However, we can start to see a pattern. After each group of three trades, the player ends up with 2  more than they started with. So after the next three trades, the player will have 1  and 6 , then 1  and 8 , and then 1  and 10 . At this point they will have at least 10 , so will have finished the level. Therefore, when a player finishes the first level, they will have 1  and 10 .

- (b) Each time the player makes the three trades, they earn 2 . Since they started with 0 , need 10  to finish the level, and $2 \times 5 = 10$, it follows that they must make the three trades 5 times. Therefore, in total, they must make $3 \times 5 = 15$ trades in the first level.



Teacher's Notes

The diagram that describes the trades and the movement between them is similar to a *flowchart* that describes a *loop* in computer science.

A loop in coding allows you to repeat instructions. In this example, you repeat a set of three trades until you have enough rubies to stop. This is an example of a *conditional loop* since the repetition continues until some condition is met.

When coding, using conditional loops can be tricky since it is possible that the stopping condition might never be met. This is called an *endless loop*. One way loops might continue forever is if the values being checked in the stopping condition never change. In our problem, we are guaranteed that the loop will stop eventually because we can show that the number of rubies increases each time it repeats.



Data Management (D)



**Take me to the
cover**



Problem of the Week

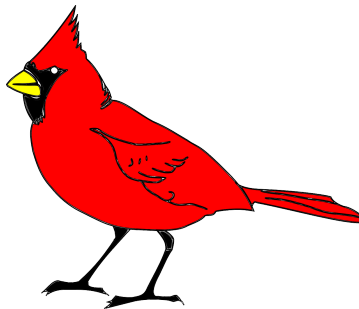
Problem A

Counting Birds

The Wildlife Centre did a bird count for one month. They counted a total of 828 birds. Their results are shown in the table below, but the count for the blue jays is missing.

Bird	Count
Sparrows	234
Chickadees	317
Blue Jays	?
Cardinals	123

- (a) How many blue jays did they count?
- (b) Did they count more sparrows and blue jays combined than chickadees and cardinals combined?





Problem of the Week

Problem A and Solution

Counting Birds

Problem

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Bird	Count
Sparrows	234
Chickadees	317
Blue Jays	?
Cardinals	123

- (a) How many blue jays did they count?
- (b) Did they count more sparrows and blue jays combined than chickadees and cardinals combined?

Solution

- (a) From the information in the table, we can find the total number of sparrows, chickadees, and cardinals is equal to $234 + 317 + 123 = 674$. Since they counted a total of 828 birds, we can find the number of blue jays by subtracting the total number of the other birds from the overall total number of birds. That is, $828 - 674 = 154$.

Therefore, they counted 154 blue jays.

- (b) They counted $234 + 154 = 388$ sparrows and blue jays combined. They counted $317 + 123 = 440$ chickadees and cardinals combined. So they counted more chickadees and cardinals combined.

Alternatively, from the given data we know that they counted $317 + 123 = 440$ chickadees and cardinals combined. Since 440 is more than half of 828, which is the total number of birds, then they must have counted more chickadees and cardinals.



Problem of the Week

Problem A

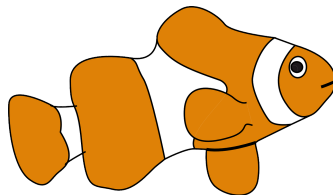
Fishing for Prizes

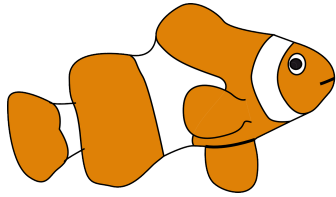
Atharv is playing a game at the school fun fair. There are 20 bowls with a different fish in each bowl. Atharv throws a ping pong ball, and if it lands in a fishbowl, then he wins the fish in that bowl. The fishbowls and the fish contained in each bowl are shown below.

betta fish	guppy	guppy	guppy	goldfish
guppy	clown fish	goldfish	betta fish	guppy
tetra	betta fish	tetra	clown fish	minnow
guppy	goldfish	tetra	guppy	betta fish

Assume that every throw lands a ball into a fishbowl and each bowl has an equal chance of being the one that the ball lands in.

- Which fish is most likely to be won by Atharv?
- Which fish is least likely to be won by Atharv?
- Which fish are equally likely to be won by Atharv?
- Is it likely that Atharv will win a clown fish?





Problem of the Week

Problem A and Solution

Fishing for Prizes

Problem

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Solution

To answer many of these questions, we should tally the number of each fish.

Type of Fish	Number
betta fish	4
guppy	7
goldfish	3
clown fish	2
tetra	3
minnow	1

We should also note that there are 20 fishbowls in total.

- (a) Since it is equally likely to land in any bowl, and since there are more bowls with a guppy than any other fish, then a guppy is the most likely fish to be won.
- (b) Since it is equally likely to land in any bowl, and since there is only 1 bowl with a minnow, then a minnow is the least likely fish to be won.
- (c) Since there are the same number of bowls that contain a goldfish as there are that contain a tetra, then it is equally likely to win a goldfish or a tetra.
- (d) Since there are 2 bowls that contain a clown fish and 18 bowls that do not contain a clown fish, it is not likely that Atharv will win a clown fish.



Problem of the Week

Problem A

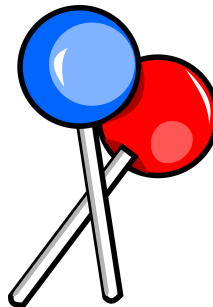
Sweet Treat

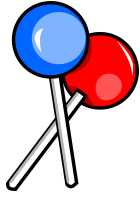
A single package of suckers contains 25 suckers of different colours: blue, red, yellow, pink, and green. There is an equal number of each colour of sucker in a package.

Miss Lolli surveys her class to find out what colour they would like to have for their special Fun Friday treat. The results of the survey are tallied as follows.

Colour	Number of Students
Blue	
Red	
Yellow	
Pink	
Green	

- (a) If Miss Lolli wants to make sure that each student receives the sucker colour they would like, how many packages of suckers does she need to buy?
- (b) After each student gets their sucker of choice on Friday, how many of each colour of sucker will she have left over?





Problem of the Week

Problem A and Solution

Sweet Treat

Problem

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Solution

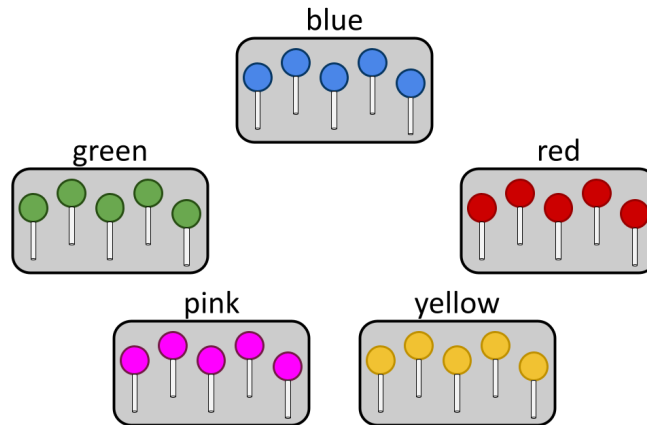
- (a) From the tally chart we know that the most popular colour is red, and 11 students selected that colour. This means Miss Lolli needs to buy enough packages to have at least 11 red suckers.

There are 25 suckers in a package, there are 5 different colours, and there are an equal number of each colour in a package. This means there are $25 \div 5 = 5$ suckers of each colour in a package.



Alternatively, we can find there are 5 suckers of each colour in a package in the following way:

If we build one pile for each colour, and we add the suckers from the package to a pile one at a time, we will find that there are 5 suckers of each colour.



Since there are 5 red suckers in a single package, then there are 10 red suckers in two packages, and 15 red suckers in three packages. This means Miss Lolli needs to buy three packages to get enough red suckers for the students.

- (b) Since there are 5 suckers of each colour in a package, then if Miss Lolli buys three packages there will be 15 suckers of each colour.

For each colour of sucker, we subtract the value of the tally from 15 to see how many will be left over of that colour:

- Blue: $15 - 4 = 11$ left over
- Red: $15 - 11 = 4$ left over
- Yellow: $15 - 2 = 13$ left over
- Pink: $15 - 7 = 8$ left over
- Green: $15 - 3 = 12$ left over

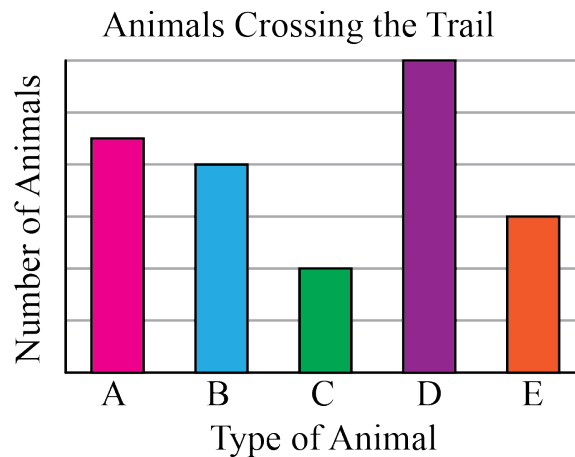


Problem of the Week

Problem A

Animals on Parade

The staff at a provincial park set up a trail camera. Over 24 hours they captured the number of foxes, chipmunks, mice, raccoons, and rabbits that crossed the trail. The bar chart below was drawn on lined paper and shows the results of their tallies, where the letters A, B, C, D, and E along the horizontal axis each represent one type of animal; however, the numbers on the vertical axis are missing.



Use the following clues to match the letters in the bar chart to the correct animal.

- There were half as many foxes as there were chipmunks
- The number of chipmunks is the median of the data. (Note: The *median* is defined below.)
- There were twice as many mice as there were raccoons.
- There are more rabbits than raccoons.

NOTE: The *Median* of a data set refers to the middle number after the numbers have been arranged in order. If a data set has an even number of values, then there are two “middle numbers”. In this case we calculate the sum of the two numbers and divide by 2 to determine the median.



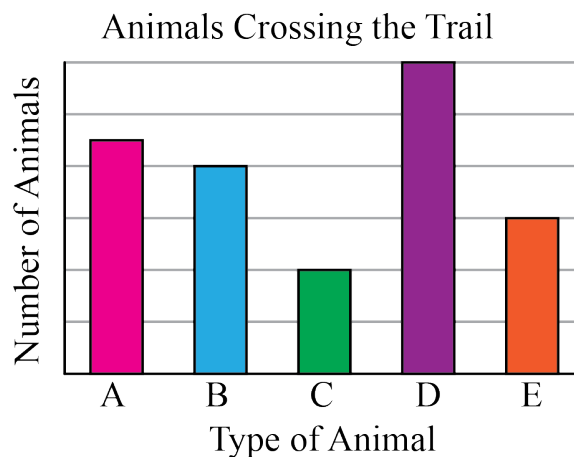
Problem of the Week

Problem A and Solution

Animals on Parade

Problem

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Solution

If the bars were ordered from shortest to tallest, then the bar labelled **B** would be in the middle. Thus, the bar labelled **B** represents the middle number of the data. Since the number of chipmunks is the median (middle) of the data, then bar **B** represents the number of chipmunks.

Since there were half as many foxes as there were chipmunks, then the bar representing the number of foxes must be half as tall as bar **B**. Since bar **C** is half the height of bar **B**, bar **C** represents the number of foxes.

Since there are more rabbits than raccoons, and there are twice as many mice as raccoons, then the shortest bar of the remaining bars **A**, **D**, and **E** must represent the number of raccoons. Therefore, bar **E** represents the number of raccoons. Since bar **D** is twice the height of bar **E** and there were twice as many mice as there were raccoons, bar **D** represents the number of mice.

Bar **A** is the only one not yet connected to an animal. This means that bar **A** must represent the number of rabbits. The height of bar **A** indicates that there are more rabbits than raccoons, which matches the last clue. Note that we did not need the last clue in any of the earlier logic, but it is a good check for us and it is important that the last clue is actually true for consistency.



Problem of the Week

Problem A

Counting Kits

Squirrels have babies twice each year, usually in March or April and again in July or August. Baby squirrels are called kits and a group of kits in one birth is called a litter.

The table below shows the number of kits a squirrel had over four years.

Month and Year of Birth	Number of Kits in the Litter
March 2020	4
July 2020	8
April 2021	6
August 2021	3
March 2022	5
August 2022	4
April 2023	6
July 2023	4

What is the average (mean) number of kits the squirrel had in a litter?





Problem of the Week

Problem A and Solution

Counting Kits

Problem

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March 2020	4
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August 2021	3
March 2022	5
August 2022	4
April 2023	6
July 2023	4

What is the average (mean) number of kits the squirrel had in a litter?

Solution

We can calculate the average by finding the total kits born and then dividing the total by the number of litters.

The total number of kits born was $4 + 8 + 6 + 3 + 5 + 4 + 6 + 4 = 40$.

Since there were 8 litters, the average number of kits born in a litter was $40 \div 8 = 5$.

Calculating the average can also be thought of as a fair share problem. In this case, we want to evenly distribute 40 kits over 8 litters. We could arrange 40 tokens so that there is an equal number of tokens in each of 8 piles. We could also start with stacks of blocks representing the number of kits born in each litter. Then we move blocks, one at a time, from the higher stacks to the lower stacks until all the stacks have the same number of blocks.



Geometry & Measurement (G)

**Take me to the
cover**



Problem of the Week

Problem A

Dog Walking

Petra walks his dog once a day. Most days when Petra walks his dog, he takes a route that is $3\frac{1}{2}$ km long. When it is raining, he does a shorter walk which is only 2 km long.

One week it rained for 3 days and did not rain on the other 4 days. How far did Petra walk his dog that week?





Problem of the Week

Problem A and Solution

Dog Walking

Problem

Petra walks his dog once a day. Most days when Petra walks his dog, he takes a route that is $3\frac{1}{2}$ km long. When it is raining, he does a shorter walk which is only 2 km long.

One week it rained for 3 days and did not rain on the other 4 days. How far did Petra walk his dog that week?

Solution

On each of the 3 days it rained, Petra walked 2 km for a total of $2 + 2 + 2 = 6$ km.

On each of the 4 days it did not rain, Petra walked $3\frac{1}{2}$ km.

We know that $3\frac{1}{2}$ is the same as $3 + \frac{1}{2}$, so over four days, the total distance Petra walked is equal to $3 + \frac{1}{2} + 3 + \frac{1}{2} + 3 + \frac{1}{2} + 3 + \frac{1}{2}$.

Collecting the whole numbers and the fractions, we can rewrite this as

$$3 + 3 + 3 + 3 + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2}.$$

Since $\frac{1}{2} + \frac{1}{2} = 1$, we can rewrite this as $3 + 3 + 3 + 3 + 1 + 1 = 14$ km.

Alternatively, to calculate the distance Petra walked on the days it did not rain, we can add $3\frac{1}{2} + 3\frac{1}{2} = 7$ km which is how far Petra walked in two days. So he walked twice as far in four days, which is $7 \times 2 = 14$ km.

So the total distance Petra walked that week is $6 + 14 = 20$ km.

Alternatively, we can do the calculation in metres.

Since we know that 1 km is equal to 1000 m, then $\frac{1}{2}$ km is equal to 500 m.

So $3\frac{1}{2}$ km is equal to $3 \times 1000 + 500 = 3500$ m and 2 km is equal to $2 \times 1000 = 2000$ m.

This means the total distance Petra walked is equal to

$2000 + 2000 + 2000 + 3500 + 3500 + 3500 + 3500 = 20000$ m, which is 20 km.



Problem of the Week

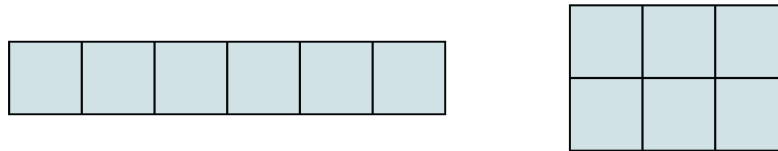
Problem A

Rearranging Rectangles

Ravi arranges identical square tiles to form rectangles using the following rules:

1. Tiles must line up exactly without any gaps or overlaps.
2. The width of each rectangle must be larger than its height.

Using 6 tiles, Ravi can form two different rectangles. The first has a width of 6 and a height of 1, and the second has a width of 3 and a height of 2, as shown.



- (a) Draw all the rectangles Ravi can form with 15 square tiles.
- (b) Draw all the rectangles Ravi can form with 24 square tiles.
- (c) Draw all the rectangles Ravi can form with 17 square tiles.
- (d) Can Ravi form more rectangles with 24 square tiles or 35 square tiles?
Justify your answer.
- (e) **Challenge:** Ravi has some number of tiles less than 100 and is able to form only one rectangle. What is the largest number of tiles that Ravi could have?





Problem of the Week

Problem A and Solution

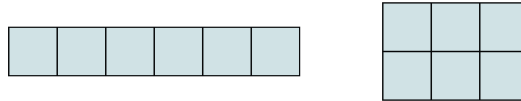
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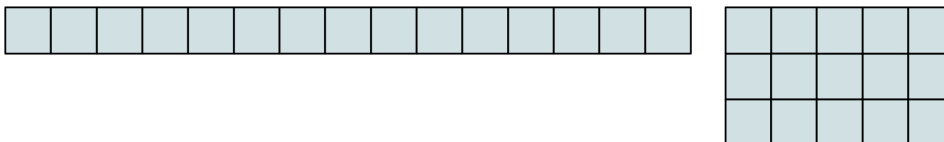
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Justify your answer.
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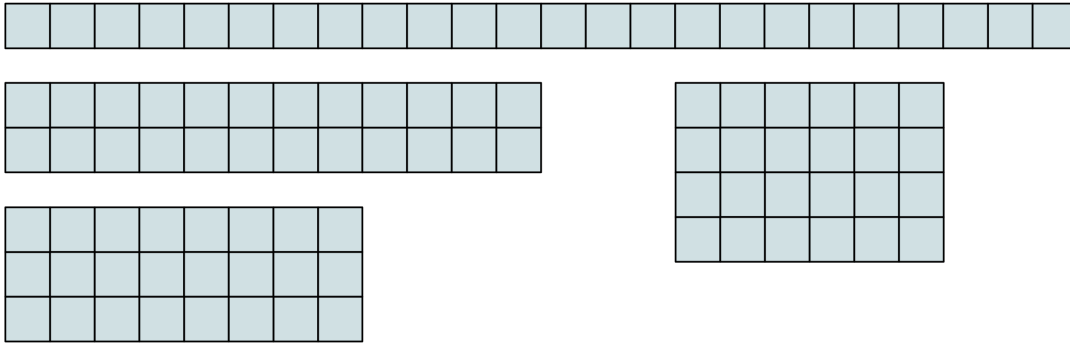
Solution

- (a) There are two possible rectangles that Ravi can form with 15 tiles.
They have dimensions 15×1 and 5×3 .





- (b) There are four possible rectangles that Ravi can form with 24 tiles. They have dimensions 24×1 , 12×2 , 8×3 , and 6×4 .



- (c) There is only one possible rectangle that Ravi can form with 17 tiles. It has dimensions 17×1 .



- (d) There are two possible rectangles that Ravi can form with 35 tiles. They have dimensions 35×1 and 7×5 . From part (b) we know that Ravi can form four rectangles with 24 tiles. So Ravi can form more rectangles with 24 square tiles than with 35 square tiles.

Another way to justify the answer is to notice the relationship between the number of rectangles Ravi can form and the number of whole numbers that are *divisors* of the number of tiles we start with in each case. Divisors are the whole numbers that divide exactly into the number of tiles that we start with. The whole number 1 is a divisor of every whole number. A whole number is always a divisor of itself.

There are 4 whole numbers that are divisors of 15: 1, 3, 5, 15, and Ravi can form 2 rectangles.

There are 8 whole numbers that are divisors of 24: 1, 2, 3, 4, 6, 8, 12, 24, and Ravi can form 4 rectangles.

There are 2 whole numbers that are divisors of 17: 1, 17, and Ravi can form 1 rectangle.

Generally, the more divisors a number has, the more rectangles Ravi can form with that number of tiles. Specifically, if there are an even number of divisors, Ravi can form half that number of rectangles. Note that if there are an odd number of divisors, then one of the rectangles Ravi can form will actually be a square. However, according to Ravi's rules, the width of each rectangle must be larger than its height, so squares are not allowed.

Since there are 4 whole numbers that are divisors of 35: 1, 5, 7, 35, then we predict that Ravi can form 2 rectangles. This means Ravi can form more rectangles with 24 tiles than with 35 tiles.

- (e) One way to figure out the largest number of tiles, less than 100, that can only form one rectangle is to look for the largest number that has only two whole numbers that are divisors of it: 1 and itself. Working backwards, we know that 99 is divisible by 3, and 98 is divisible by 2. However, 97 is not divisible by any whole numbers except 1 and 97. So the largest number of tiles that Ravi could have is 97.



Teacher's Notes

Rearranging identical square tiles into rectangles is a way to find the *factors* of a whole number. A factor of a whole number is another name for a divisor of a whole number.

A *prime number* is a whole number greater than 1 that has exactly two whole number factors: 1 and itself. So 2 is a prime number, but 4 is not a prime number. In our problem of finding the number of rectangles that can be formed by identical square tiles, a prime number of tiles will always result in one rectangle that has a height of 1 and a width equal to the number of tiles.

A *composite number* is a whole number greater than 1 that has more than two whole number factors. A composite number that is a perfect square has an odd number of whole number factors. Most factors come in pairs of two different numbers, where the two numbers multiplied together equal the number you are factoring. With a perfect square, one of the factors of the number is its square root. The square root of a number n is a number that when multiplied by itself is equal to n . So the factor that is equal to the square root does not have a different number to pair up with, which results in an odd number of factors.

In our problem, if we had a number of tiles that is a perfect square of a prime number, this would result in exactly one rectangle, since the width must be larger than its height. In these cases, we could form a square out of the tiles, where the length of the side of the square is equal to the square root of the number. An example of this would be the number 49. It has three whole number factors: 1, 7, and 49, and you can only form one rectangle (with dimensions 1×49) and one square (with dimensions 7×7) out of 49 tiles.



Problem of the Week

Problem A

Bean There; Done That

Suppose you want to make a four-bean salad using green beans, wax beans, kidney beans, and garbanzo beans. When you weigh the ingredients you notice the following:

- In total, the mass of all the beans in the salad is 1 kg.
- The sum of the mass of the garbanzo beans and the mass of the green beans make up half of the total mass of the beans in the salad.
- The masses of the garbanzo beans and the green beans are the same.
- The mass of the wax beans is 235 g.

What is the mass of each type of bean in the salad? Justify your answer.





Problem of the Week

Problem A and Solution

Bean There; Done That

Problem

Suppose you want to make a four-bean salad using green beans, wax beans, kidney beans, and garbanzo beans. When you weigh the ingredients you notice the following:

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- The masses of the garbanzo beans and the green beans are the same.
- The mass of the wax beans is 235 g.

What is the mass of each type of bean in the salad? Justify your answer.

Solution

We know that 1 kg equals 1000 g. Half the total mass of the beans is $1000 \div 2 = 500$ g. So the total mass of the garbanzo beans and the green beans is 500 g. Since the remaining salad is made up of the other two kinds of beans, the total mass of the wax beans and the kidney beans must also be 500 g.

Since the masses of the garbanzo beans and the green beans are the same, then each mass is equal to $500 \div 2 = 250$ g.

Since the mass of the wax beans is 235 g, then the mass of the kidney beans must be $500 - 235 = 265$ g.

In summary, the salad contains:

- 250 g of garbanzo beans
- 250 g of green beans
- 235 g of wax beans
- 265 g of kidney beans



Problem of the Week

Problem A

Tri Kids Race

A triathlon is a race that has three components. Racers first complete a swimming component, then they complete a biking component, and finally they complete a running component.

In the Tri Kids race, the racers first swim 100 m. They then travel 25 m to the bicycle area. The racers then ride their bike in a 3 km long loop back to the bicycle area. The bicycle area is right beside the track. The racers then run 3 laps of the track to finish the race. One lap of the track is 400 m.

In the Tri Kids race, what is the total distance the racers have to cover, from start to finish?





Problem of the Week

Problem A and Solution

Tri Kids Race

Problem

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In the Tri Kids race, what is the total distance the racers have to cover, from start to finish?

Solution

To calculate the total distance, we can convert all measurements to metres. The distance travelled on their bike is 3 km. Since 1 km is equal to 1000 m, then 3 km is equal to $3 \times 1000 = 3000$ m.

To calculate the distance the racers have to run, we multiply $3 \times 400 = 1200$ m.

We can add up the distances travelled in the swim, transition to the bicycle area, bike, and run. The total distance covered is: $100 + 25 + 3000 + 1200 = 4325$ m.

Alternatively, we can enumerate the distances travelled in a table:

Race Component	Distance of Component	Total Distance Travelled
Swim	100 m	100 m
Transition	25 m	125 m
Bike	3 km = 3000 m	3125 m
Lap 1	400 m	3525 m
Lap 2	400 m	3925 m
Lap 3	400 m	4325 m



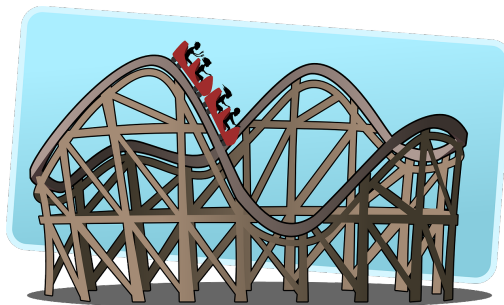
Problem of the Week

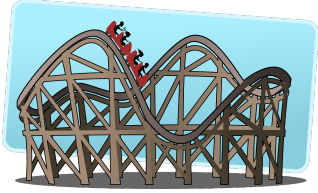
Problem A

Roller Coaster Riders

A local amusement park has many roller coasters. On the roller coaster Gargantuan, the train has 8 cars, seating 4 guests in each car at one time. A ride starts every 3 minutes.

If the roller coaster was full every time, how many people rode the Gargantuan in a half an hour from the first ride starting? Justify your answer.





Problem of the Week

Problem A and Solution

Roller Coaster Riders

Problem

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If the roller coaster was full every time, how many people rode the Gargantuan in a half an hour from the first ride starting? Justify your answer.

Solution

If there are 8 cars, and each car can hold 4 guests, then the maximum capacity of the roller coaster is $8 \times 4 = 32$. In other words, at most 32 people can ride the Gargantuan at one time.

Half an hour is equal to 30 minutes. Now we can make a table to keep track of the total number of riders, if 32 people ride the Gargantuan every 3 minutes.

Time Elapsed (minutes)	Total Number of Riders
3	32
6	64
9	96
12	128
15	160
18	192
21	224
24	256
27	288
30	320

Alternatively, we can calculate that in 30 minutes, there must be $30 \div 3 = 10$ rides completed. Since the maximum capacity of one ride is 32 people, then the maximum number of riders in 30 minutes is $32 \times 10 = 320$ people.



Problem of the Week

Problem A

Feeding the Pets

I give my dog 162 g of dog food each meal. He eats two meals per day.

I give my cat 95 g of cat food each meal. She eats one meal per day.

The dog food comes in a 5 kg bag. The cat food comes in a 2 kg bag.

If I start a new bag of food for each pet on the same day, which bag will run out first? Justify your answer.





Problem of the Week

Problem A and Solution

Feeding the Pets

Problem

I give my dog 162 g of dog food each meal. He eats two meals per day.

I give my cat 95 g of cat food each meal. She eats one meal per day.

The dog food comes in a 5 kg bag. The cat food comes in a 2 kg bag.

If I start a new bag of food for each pet on the same day, which bag will run out first? Justify your answer.

Solution

One way to determine which bag will run out first is to make a table that keeps track of how much food each pet consumes.

Since the dog eats two meals of 162 g each day, then he consumes $162 + 162 = 324$ g per day. Since $1 \text{ kg} = 1000 \text{ g}$, then $2 \text{ kg} = 2 \times 1000 = 2000 \text{ g}$ and $5 \text{ kg} = 5 \times 1000 = 5000 \text{ g}$. We will continue tracking the amount of food consumed until we reach one of these limits.

Day	Total Consumed by Cat (in g)	Total Consumed by Dog (in g)
1	95	324
2	190	648
3	285	972
4	380	1296
5	475	1620
6	570	1944
7	665	2268
8	760	2592
9	855	2916
10	950	3240
11	1045	3564
12	1140	3888
13	1235	4212
14	1330	4536
15	1425	4860
16	1520	5184

On Day 16 the dog will have consumed more than 5000 g of food, but the cat has not yet consumed 2000 g of food. So the dog food bag will run out first.

Alternatively, we could use estimation to determine which bag will run out first. Since 95 g is less than 100 g, then the cat food will last at least 20 days since $20 \times 100 = 2000$. Since 324 g is greater than 250 g, then the dog food will last less than 20 days since $20 \times 250 = 5000$.

Therefore, the cat food will last longer than the dog food, so the dog food bag will run out first.





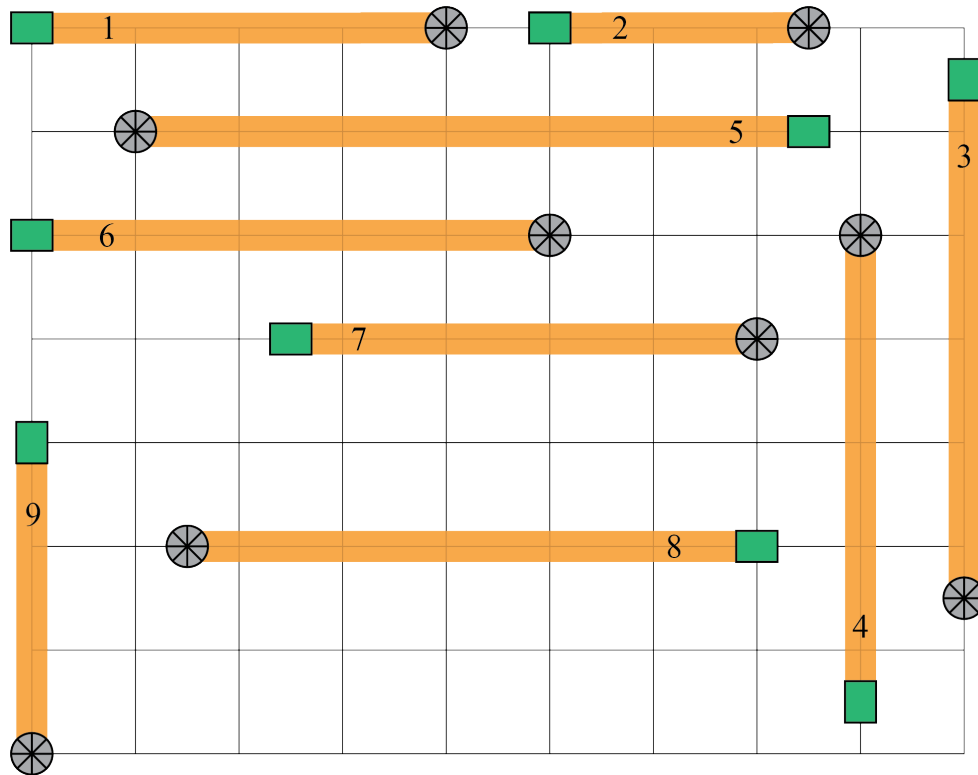
Problem of the Week

Problem A

Disc Golf Distance

Disc golf is a sport where players throw a flying disc towards a target. A disc golf course is made up of several holes, each with their own tee box and basket. Players start by standing in the tee box and throwing the disc towards the basket.

Hypatia Public School is building a new nine-hole disc golf course on their property. They drew a plan for the course on grid paper, showing the tee box , basket , and hole number for each hole. The distance between grid lines is 20 m, and the centre of each tee box and basket is placed either on a grid line, or halfway between two grid lines.



The length of a hole is the distance from the centre of the tee box to the centre of the basket. The length of a course is the sum of the lengths of all the holes, not including the distances between the holes.

Calculate the length of the disc golf course the school is planning to build.





Problem of the Week

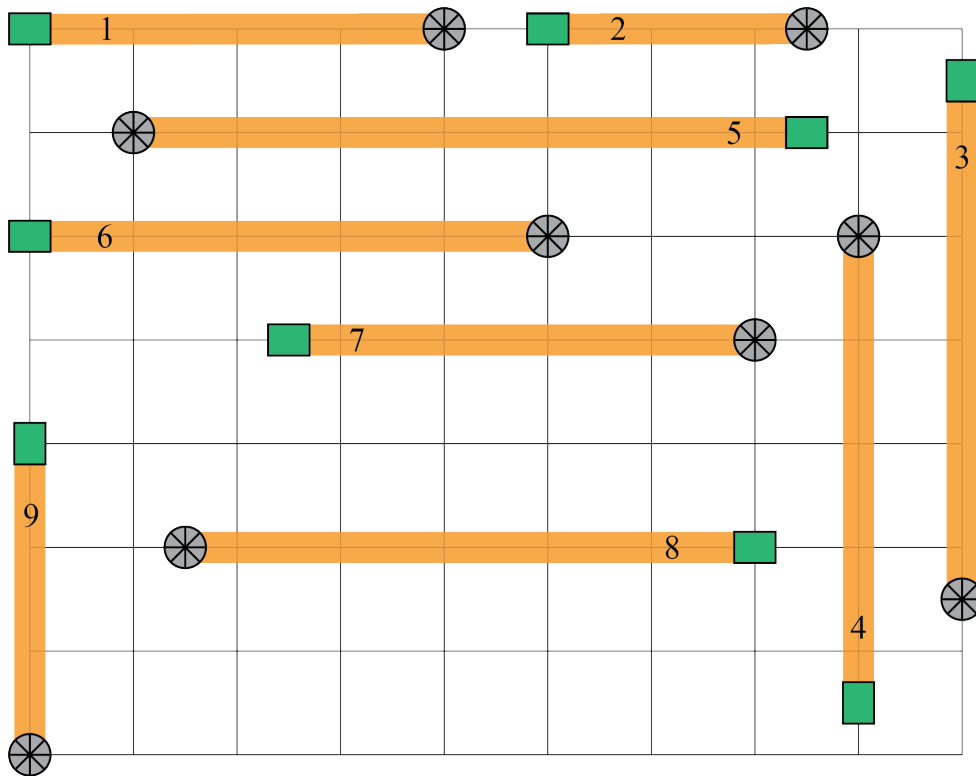
Problem A and Solution

Disc Golf Distance

Problem

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Solution

Since the distance between the grid lines represents 20 m, then the distance halfway from one grid line to the next would be 10 m. Keeping this in mind, we can calculate the length of each hole by counting the number of grid squares between its tee box and basket.

Hole Number	Number of Grid Squares Between Tee Box and Basket	Length of Hole (m)
1	four	$4 \times 20 = 80$
2	two and a half	$2 \times 20 + 10 = 50$
3	five (a half, plus 4, plus a half)	$5 \times 20 = 100$
4	four and a half	$4 \times 20 + 10 = 90$
5	six and a half	$6 \times 20 + 10 = 130$
6	five	$5 \times 20 = 100$
7	four and a half	$4 \times 20 + 10 = 90$
8	five and a half	$5 \times 20 + 10 = 110$
9	three	$3 \times 20 = 60$

So the length of the course is:

$$80 + 50 + 100 + 90 + 130 + 100 + 90 + 110 + 60 = 810 \text{ m.}$$

Alternatively, we can calculate the total number of grid squares between the tee boxes and the baskets, and then use this to calculate the length of the course.

First we add the complete squares: $4 + 2 + 4 + 4 + 6 + 5 + 4 + 5 + 3 = 37$.

Then we add the half squares, using the fact that $\frac{1}{2} + \frac{1}{2} = 1$.

$$\frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} = 1 + 1 + 1 + \frac{1}{2} = 3 + \frac{1}{2}$$

Then we add these together to obtain $37 + 3 + \frac{1}{2} = 40 + \frac{1}{2}$.

Finally we can calculate the length of the course in meters. Since $40 \times 20 = 800$, the length of the course is $800 + 10 = 810 \text{ m.}$



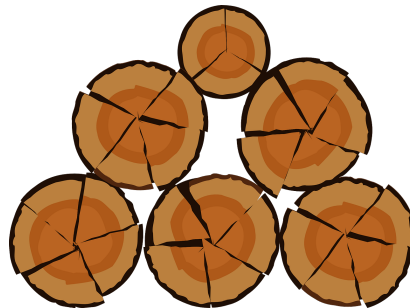
Problem of the Week

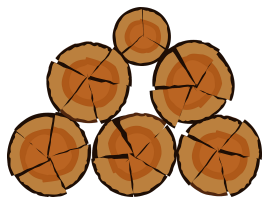
Problem A

Piling Wood

The Taylor family heats their home with wood. They recently got a large delivery of cut logs which were left in a pile on their driveway. Janelle and Alphonso carry logs from the pile and stack them in the woodshed. In one trip, Janelle can carry 4 logs at a time and Alphonso can carry 3 logs at a time. They each take the same amount of time to carry logs from the pile, stack them in the woodshed, and walk back to the pile.

- If they each take as many logs as they can on each trip from the pile to the woodshed, how many logs have Janelle and Alphonso stacked after making 10 trips from the pile to the woodshed?
- When Janelle and Alphonso started, there were 200 logs in the pile. How many trips from the pile to the woodshed do they have to take in order to move all the logs?
- It takes approximately 30 seconds for Janelle and Alphonso to carry logs from the pile, stack them in the woodshed, and return to the pile. Approximately how many minutes does it take them to stack all of the logs in part (b)?
- On the last trip, Janelle and Alphonso carry the same number of logs. If they each take as many logs as they can on their earlier trips, how many logs do they each carry on the last trip?





Problem of the Week

Problem A and Solution

Piling Wood

Problem

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- On the last trip, Janelle and Alphonso carry the same number of logs. If they each take as many logs as they can on their earlier trips, how many logs do they each carry on the last trip?

Solution

- In total, Janelle and Alphonso carry $4 + 3 = 7$ logs per trip from the log pile to the woodshed. This means they carry $7 \times 10 = 70$ logs in 10 trips.
- To figure out how many trips it takes to move all the logs, we could skip count by 7s until we get to or pass 200:

7, 14, 21, 28, 35, 42, 49, 56, 63, 70, 77, 84, 91, ...

Then we would count how many 7s it takes to get past 200 and we would see that it is 29.

Alternatively, we know from part (a) that after 10 trips, they will have moved 70 logs. So, after 20 trips they will have moved 140 logs, and after 30 trips they will have moved 210 logs. If we count backwards from 210, we can



determine that after 29 trips they will have moved 203 logs, and after 28 trips they will have moved 196 logs. Therefore, we can conclude that it takes 29 trips to move all the logs.

Another way to calculate this is to divide 200 by 7 to obtain 28 remainder 4. Since there is a remainder after 28 trips, then it will take one more trip to move all the logs. Therefore, we can conclude that it takes 29 trips to move all the logs.

- (c) Since each trip takes approximately 30 seconds, then 2 trips take approximately 1 minute. This means that 28 trips would take approximately 14 minutes. Therefore, 29 trips would take approximately 14 and a half minutes.
- (d) After 28 trips, they have moved $28 \times 7 = 196$ logs. This means there are $200 - 196 = 4$ logs left for their last trip. Since Janelle and Alphonso carry the same number of logs on their last trip, then they each must carry 2 logs.



Problem of the Week

Problem A

Charity Chocolate Cake

Sam's class is having a bake sale. Sam wants to make cakes with his mom for the fundraiser. Their oven can only fit one cake at a time.

There are four steps to make one cake.

Step 1: Mix the batter. This takes 10 minutes.

Step 2: Bake the cake. This takes 35 minutes.

Step 3: Let the cake cool for 15 minutes.

Step 4: Frost the cake. This takes 10 minutes.

- (a) How long does it take to make one cake from start to finish?
- (b) Sam wants to bake two cakes for the bake sale. He starts making the second cake immediately after frosting the first cake. If they need to leave for the bake sale by 4:00 p.m., what is the latest possible time that Sam can start making the first cake?

EXTENSION: You may have noticed that you can make two cakes faster by mixing the batter for the second cake while the first cake is baking, and putting the second cake in the oven immediately after taking the first cake out. Then you can cool and frost the first cake while the second cake is baking. How long would it take from the time you start mixing the batter for the first cake until you finish frosting the second cake with this plan?





Problem of the Week

Problem A and Solution

Charity Chocolate Cake

Problem

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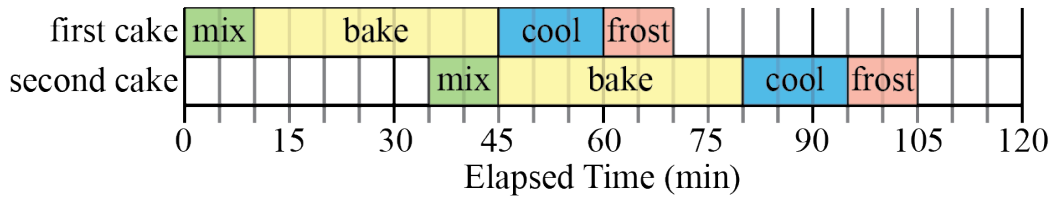
Solution

- (a) The total time to bake one cake is $10 + 35 + 15 + 10 = 70$ minutes, or 1 hour and 10 minutes.
- (b) Since Sam completely finishes baking and frosting one cake before starting the second cake, it will take twice as long to make two cakes. This means it will take $70 \times 2 = 140$ minutes, or 2 hours and 20 minutes. Now we have to count time backwards from 4:00 p.m. to determine when Sam needs to start making the first cake. Two hours before 4:00 p.m. is 2:00 p.m., and 20 minutes before 2:00 p.m. is 1:40 p.m. Therefore, Sam needs to start making the first cake by 1:40 p.m. at the latest.

Another way to calculate what time it is 140 minutes before 4:00 p.m. is to use a clock and move the hands back in time. There are some virtual options for this including <https://toytheater.com/clock/>.



EXTENSION: To calculate the time it would take to make two cakes with this plan, you could make a timeline as shown below:



Counting the time from the start until the second cake is finished, we can see it takes 105 minutes, or 1 hour and 45 minutes. Note that the second cake batter can be mixed any time during the 35 minutes the first cake is baking.

Alternatively, you can record the time you are saving when doing something with one cake while the other cake is baking. For the first cake, you're saving the time for cooling and frosting, which is $15 + 10 = 25$ minutes. For the second cake, you're saving the time for mixing the batter, which is 10 minutes. Therefore, in total, you're saving $25 + 10 = 35$ minutes. Since our answer from part (b) was 140 minutes, then with our faster plan we can bake the two cakes in $140 - 35 = 105$ minutes, or 1 hour and 45 minutes.



Teacher's Notes

The extension problem is an example of how *pipelining* can improve production. The basic idea behind pipelining is to break up a task into smaller parts, and complete the smaller parts of separate tasks at the same time (i.e. in parallel).

In this example, mixing the second cake's batter takes place at the same time as the first cake is baking. As well, the first cake is cooling and being frosted at the same time as the second cake is baking. Without pipelining, it takes 2 hours and 20 minutes to finish both cakes. With pipelining, it takes 1 hour and 45 minutes to finish both cakes.

Pipelining is used in lots of places such as assembly lines. It is also used in the design of computer chips to increase the processing speed of your digital devices.

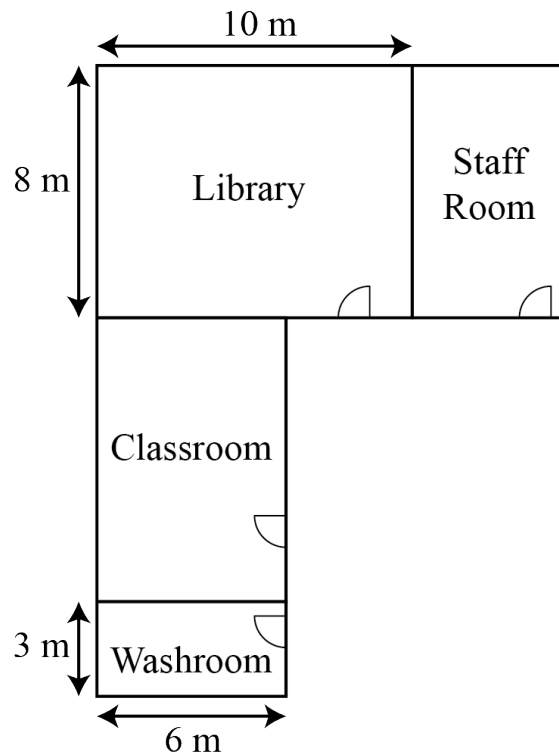


Problem of the Week

Problem A

Room to Grow

Ada Lovelace Public School needs to add a new wing to its main building that includes a classroom, library, staff room, and washroom. The plans for the addition are shown, with all new rooms being rectangular; however, some of the measurements are missing.



Fortunately, the contractor remembered the following extra details:

The length of the classroom is three times the width of the washroom.
The width of the staff room is half the length of the library.

When describing the dimensions of a rectangle, the contractor uses *length* to describe the longer side, and *width* to describe the shorter side.

- What are the missing dimensions of the classroom and the staff room?
- What is the total floor area of the four new rooms?



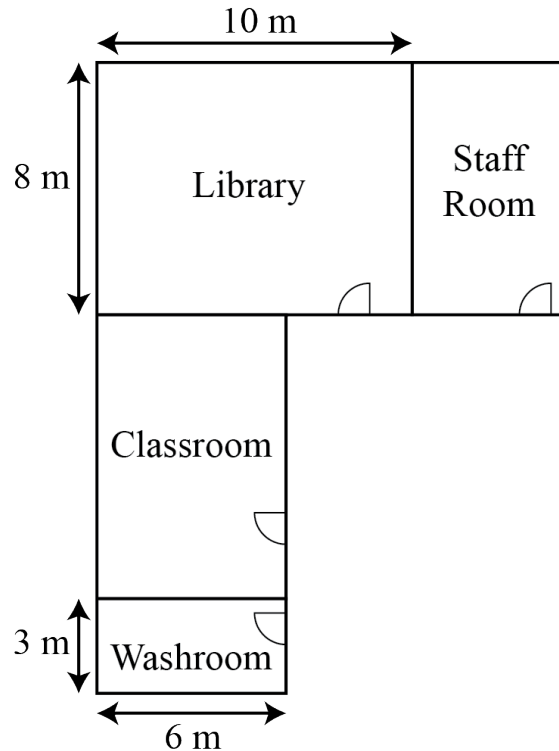
Problem of the Week

Problem A and Solution

Room to Grow

Problem

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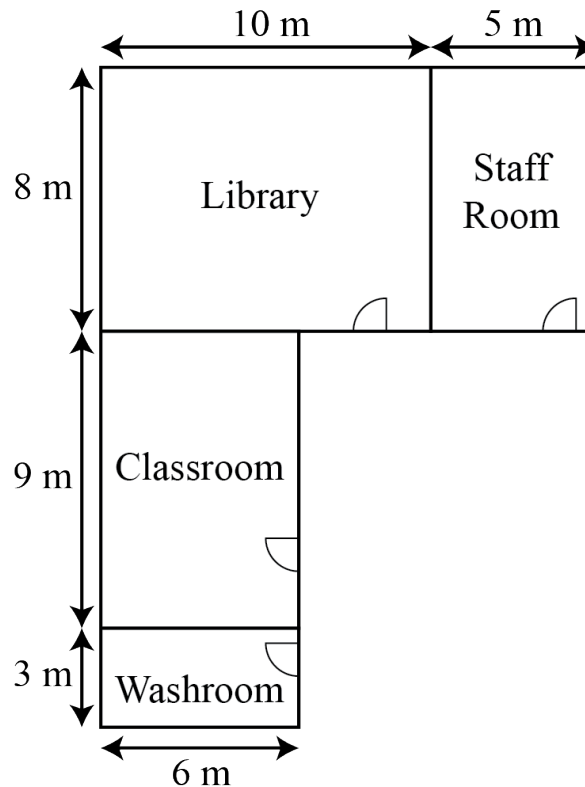
When describing the dimensions of a rectangle, the contractor uses *length* to describe the longer side, and *width* to describe the shorter side.

- What are the missing dimensions of the classroom and the staff room?
- What is the total floor area of the four new rooms?



Solution

- (a) Since the length of the classroom is three times the width of the washroom, its length is $3 \times 3 = 9$ m. Since the width of the staff room is half the length of the library, its width is $10 \div 2 = 5$ m. These dimensions are shown in the diagram.



- (b) There are many ways to determine the total floor area.

Solution 1: We calculate the area of each room and add those areas together.

- The area of the washroom is $6 \times 3 = 18$ m².
- The area of the classroom is $9 \times 6 = 54$ m².
- The area of the library is $8 \times 10 = 80$ m².
- The area of the staff room is $8 \times 5 = 40$ m².

Therefore, the total floor area is $18 + 54 + 80 + 40 = 192$ m².

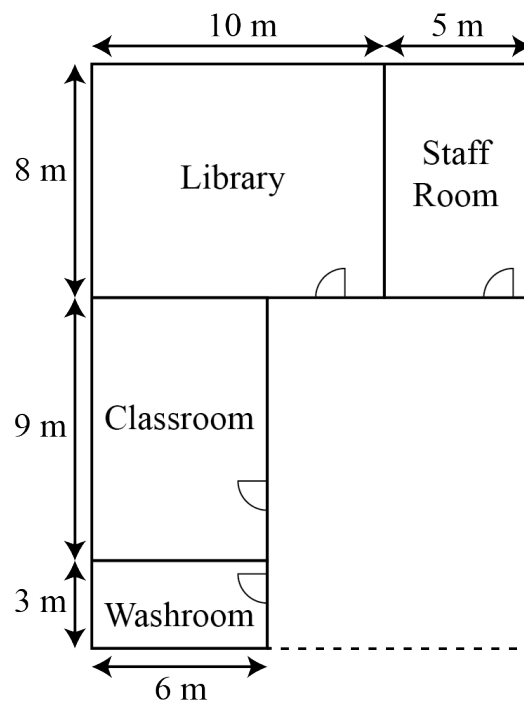
Solution 2: We calculate the area of the classroom based on the area of the washroom, and calculate the area of the staff room based on the area of the library. Since the length of the classroom is three times the width of the washroom, but the width of the classroom is the same as the length of the



washroom, the area of the classroom is three times the area of the washroom. We can imagine that we could put three washrooms side-by-side and they would fill in the same area as the classroom. Since the area of the washroom is $6 \times 3 = 18 \text{ m}^2$, it follows that the area of the classroom is $18 \times 3 = 54 \text{ m}^2$. Similarly, the area of the staff room would fill half of the area of the library. Since the area of the library is $8 \times 10 = 80 \text{ m}^2$, it follows that the area of the staff room is $80 \div 2 = 40 \text{ m}^2$.

Therefore, the total floor area is $18 + 54 + 80 + 40 = 192 \text{ m}^2$.

Solution 3: We square off the diagram to create a large rectangle, as shown.



This large rectangle measures $10 + 5 = 15 \text{ m}$ across the top and $8 + 9 + 3 = 20 \text{ m}$ along the side. The total area of this rectangle is then $15 \times 20 = 300 \text{ m}^2$.

However, there is a section of the large rectangle that is not part of the floor plan, and hence its area needs to be subtracted. That section is a rectangle with length $3 + 9 = 12 \text{ m}$ and width $15 - 6 = 9 \text{ m}$. The area of the section is then $12 \times 9 = 108 \text{ m}^2$.

Therefore, the total floor area is equal to $300 - 108 = 192 \text{ m}^2$.



Problem of the Week

Problem A

Scrabble Words

Scrabble™ is a game where players place tiles containing individual letters on a board to form words. The board is divided up into squares, and each letter in a word is placed in adjacent squares in a row (reading from left to right) or a column (reading from the top down). Each letter has a point value, and a player’s score for placing a word on the board is the sum of the point values for the letters in the word. The point values for each letter are in the following table.

Letter	A	B	C	D	E	F	G	H	I	J	K	L	M
Value	1	3	3	2	1	4	2	4	1	8	5	1	3
Letter	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
Value	1	1	3	10	1	1	1	1	4	4	8	4	10

In the Scrabble board shown, the coloured squares have special values.

Blue squares, marked **B** on the board, are double letter squares, which means if you put a tile on that square, then the letter is worth twice as many points.

Grey squares, marked **G** on the board, are triple letter squares, which means if you put a tile on that square, then the letter is worth three times as many points.

Red squares, marked **R** on the board, are double word squares, which means if you put a tile on that square, then the whole word is worth twice as many points.

Orange squares, marked **O** on the board, are triple word squares, which means if you put a tile on that square, then the whole word is worth three times as many points.

Double and triple letter scores are calculated before the double and triple word scores.

G				G	
	R				
		R			B
			R		
G				R	
		B			O

- (a) How many points would the word EXACT be worth if you put the first letter in the first row and second column, and place the word horizontally across the board?
- (b) How many points would the word DEBUT be worth if you put the first letter in the second row and second column, and place the word vertically on the board?
- (c) How many points would the word CHEF be worth if you put the first letter of the word in the last column and third row, and place the word vertically on the board?



Problem of the Week

Problem A and Solution

Scrabble Words

Problem

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G				G	
	R				
		R			B
			R		
G				R	
		B			O

- How many points would the word EXACT be worth if you put the first letter in the first row and second column, and place the word horizontally across the board?
- How many points would the word DEBUT be worth if you put the first letter in the second row and second column, and place the word vertically on the board?
- How many points would the word CHEF be worth if you put the first letter of the word in the last column and third row, and place the word vertically on the board?



Solution

- (a) If we place the letters for EXACT horizontally on the board starting with the letter E in the first row and second column, then the letter C will be on a grey square. This means that letter is worth three times its normal value. That is, the letter C is worth $3 \times 3 = 9$ points. So the score for this word is $1 + 8 + 1 + 9 + 1 = 20$ points.
- (b) If we place the letters for DEBUT vertically down the board starting with the D in the second row and second column, then the word covers a red square. This means the word is worth twice its normal value. The normal score for the word would be $2 + 1 + 3 + 1 + 1 = 8$. The red square means that the word would be worth twice as much, so the score for the word at this location on the board is $2 \times 8 = 16$ points.
- (c) If we place the letters for CHEF vertically down the board starting with the letter C in the last column and third row, then the letter C will be on a blue square. This means that letter is worth twice its normal value. That is, in this placement of the word, the letter C is worth $2 \times 3 = 6$ points. The word also covers an orange square, which means the word is worth three times its normal value. We calculate the score including the double value of the letter C before applying the triple word score. The score for the word without considering the triple word square, but including its double letter score, is $6 + 4 + 1 + 4 = 15$. When we consider the triple word square, the result is $3 \times 15 = 45$ points.



Teacher's Notes

When we apply the rule that double and triple letter scores are calculated before the double and triple word scores, we are enforcing an *order of operations* to our calculations.

We often use the mnemonic **BEDMAS** to describe the standard order of operations for mathematical operators. This stands for **B**rackets, **E**xponents, **D**ivision and **M**ultiplication in the order that they appear, and **A**ddition and **S**ubtraction in the order that they appear.

We use brackets in places where we want some operation to take precedence over another operation that would otherwise happen first according to BEDMAS. For example, in part (b) of this problem, we want to add the letter values together before we apply the double word score. We could calculate the result like this:

$$(2 + 1 + 3 + 1 + 1) \times 2 = (8) \times 2 = 16$$

Without the brackets the calculation

$$2 + 1 + 3 + 1 + 1 \times 2$$

would equal

$$2 + 1 + 3 + 1 + 2 = 9$$

since according to BEDMAS the multiplication is done before the addition.

In part (c), we need to multiply the point value of the letter C by 2 since it is on a double letter square. Then we add the rest of the letter values together before multiplying that sum by 3. So we could calculate the result for part (c) like this:

$$(2 \times 3 + 4 + 1 + 4) \times 3 = (6 + 4 + 1 + 4) \times 3 = (15) \times 3 = 45$$

Note that we do not need brackets around 2×3 since, according to BEDMAS, the multiplication will be done before the addition in the calculation. However, sometimes we use brackets (even when they are not required) to make calculations clearer. To emphasize that the double letter score must be calculated before the sum of the letters is calculated, it may be easier to understand the calculation for part (c) if we used this expression:

$$((2 \times 3) + 4 + 1 + 4) \times 3$$



Number Sense (N)

**Take me to the
cover**



Problem of the Week

Problem A

What Number Am I?

I am a 4-digit number.

My ones digit plus my thousands digit is equal to my tens digit plus my hundreds digit.

My tens digit is twice as much as my thousands digit.

My ones digit is the largest single-digit whole number.

My thousands digit is 4.

What number am I?





Problem of the Week

Problem A and Solution

What Number Am I?

Problem

I am a 4-digit number.

My ones digit plus my thousands digit is equal to my tens digit plus my hundreds digit.

My tens digit is twice as much as my thousands digit.

My ones digit is the largest single-digit whole number.

My thousands digit is 4.

What number am I?

Solution

We are given that the ones digit is the largest single-digit whole number. Since the largest single-digit whole number is 9, the ones digit must be a 9.

We are also given that the thousands digit is a 4. Since the tens digit is twice as much as the thousands digit, then the tens digit must be $2 \times 4 = 8$.

Also, the sum of the ones digit and the thousands digit is $9 + 4 = 13$. Thus, the sum of the hundreds digit and the tens digit must be 13.

Since the tens digit is 8, then the hundreds digit must be $13 - 8 = 5$.

Thus, the thousands digit is 4, the hundreds digit is 5, the tens digit is 8, and the ones digit is 9. Therefore, the number is 4589.



Problem of the Week

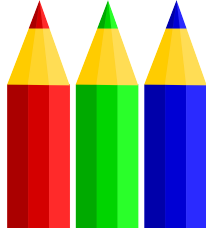
Problem A

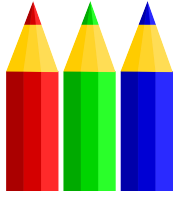
Creating an Art Kit

A teacher at Sioux Mountain School wants to purchase items for art kits to be used in their classroom. Below is a list of the items in one kit, along with the price for each item. All prices include tax.

Pencil crayons	\$1.75
Sketch book	\$2.25
Watercolour paints	\$3.15
Glue	\$1.10
Scissors	\$1.70

If the teacher has a budget of \$80 for art supplies, then estimate how many kits they can make. Justify your answer.





Problem of the Week

Problem A and Solution

Creating an Art Kit

Problem

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Sketch book	\$2.25
Watercolour paints	\$3.15
Glue	\$1.10
Scissors	\$1.70

If the teacher has a budget of \$80 for art supplies, then estimate how many kits they can make. Justify your answer.

Solution

One way to estimate how much each kit will cost is to round the price of each item to the nearest dollar:

Pencil crayons	\$2
Sketch book	\$2
Watercolour paints	\$3
Glue	\$1
Scissors	\$2

Thus, each art kit will cost approximately $\$2 + \$2 + \$3 + \$1 + \$2 = \10 .

With a budget of \$80 and since $80 \div 10 = 8$, we estimate they can make 8 art kits.



Teacher's Notes

When we use estimation we need to realize that our estimated result is an approximation and not an absolute guarantee of a correct answer. We should always consider a margin of error for our estimations.

For example, in this problem if you calculate the actual cost of an art kit, it comes to:

$$1.75 + 2.25 + 3.15 + 1.10 + 1.70 = \$9.95$$

In this case, our estimation gave us the actual number of art kits that the teacher can purchase. However, if the cost of the paints was actually \$3.25, instead of \$3.15, then the actual cost of an art kit would be \$10.05, but our estimate would still be \$10. This is a reasonable estimation, but the actual cost means the teacher does not have the budget for 8 kits, but rather for only 7 kits.

Since we used estimation to predict the number of kits that could be purchased, it would be more accurate to give an answer of 8 kits, plus or minus 1 kit. This factors in the margin of error.



Problem of the Week

Problem A

Dog Walking

Petra walks his dog once a day. Most days when Petra walks his dog, he takes a route that is $3\frac{1}{2}$ km long. When it is raining, he does a shorter walk which is only 2 km long.

One week it rained for 3 days and did not rain on the other 4 days. How far did Petra walk his dog that week?





Problem of the Week

Problem A and Solution

Dog Walking

Problem

Petra walks his dog once a day. Most days when Petra walks his dog, he takes a route that is $3\frac{1}{2}$ km long. When it is raining, he does a shorter walk which is only 2 km long.

One week it rained for 3 days and did not rain on the other 4 days. How far did Petra walk his dog that week?

Solution

On each of the 3 days it rained, Petra walked 2 km for a total of $2 + 2 + 2 = 6$ km.

On each of the 4 days it did not rain, Petra walked $3\frac{1}{2}$ km.

We know that $3\frac{1}{2}$ is the same as $3 + \frac{1}{2}$, so over four days, the total distance Petra walked is equal to $3 + \frac{1}{2} + 3 + \frac{1}{2} + 3 + \frac{1}{2} + 3 + \frac{1}{2}$.

Collecting the whole numbers and the fractions, we can rewrite this as

$$3 + 3 + 3 + 3 + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2}$$

Since $\frac{1}{2} + \frac{1}{2} = 1$, we can rewrite this as $3 + 3 + 3 + 3 + 1 + 1 = 14$ km.

Alternatively, to calculate the distance Petra walked on the days it did not rain, we can add $3\frac{1}{2} + 3\frac{1}{2} = 7$ km which is how far Petra walked in two days. So he walked twice as far in four days, which is $7 \times 2 = 14$ km.

So the total distance Petra walked that week is $6 + 14 = 20$ km.

Alternatively, we can do the calculation in metres.

Since we know that 1 km is equal to 1000 m, then $\frac{1}{2}$ km is equal to 500 m.

So $3\frac{1}{2}$ km is equal to $3 \times 1000 + 500 = 3500$ m and 2 km is equal to $2 \times 1000 = 2000$ m.

This means the total distance Petra walked is equal to

$2000 + 2000 + 2000 + 3500 + 3500 + 3500 + 3500 = 20000$ m, which is 20 km.



Problem of the Week

Problem A

Banana Bonanza

Three monkeys found a tree that has 54 bananas growing on it. Each monkey eats two bananas a day from the tree, and no other animals eat any of the bananas.

- (a) How many bananas will be left on the tree at the end of the first day?
- (b) If they start eating from the tree on a Monday, on what day of the week will they eat the last banana?





Problem of the Week

Problem A and Solution

Banana Bonanza

Problem

Three monkeys found a tree that has 54 bananas growing on it. Each monkey eats two bananas a day from the tree, and no other animals eat any of the bananas.

- (a) How many bananas will be left on the tree at the end of the first day?
- (b) If they start eating from the tree on a Monday, on what day of the week will they eat the last banana?

Solution

- (a) Since each monkey eats 2 bananas and there are 3 monkeys, this means $2 + 2 + 2$ or $2 \times 3 = 6$ bananas are eaten from the tree each day. Since the tree started with 54 bananas, this means it will have $54 - 6 = 48$ bananas left at the end of the first day.
- (b) We can make a table to keep track of how many bananas are left on the tree at the end of each day, knowing that the monkeys eat 6 bananas a day. From part (a), we know that there are 48 bananas left at the end of the first Monday.

Day	Bananas Left at End of Day
Monday	48
Tuesday	42
Wednesday	36
Thursday	30
Friday	24
Saturday	18
Sunday	12
Monday	6
Tuesday	0

From this table, we can see that they will eat the last banana on a Tuesday.



Problem of the Week

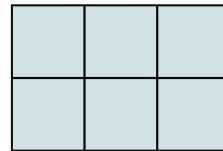
Problem A

Rearranging Rectangles

Ravi arranges identical square tiles to form rectangles using the following rules:

1. Tiles must line up exactly without any gaps or overlaps.
2. The width of each rectangle must be larger than its height.

Using 6 tiles, Ravi can form two different rectangles. The first has a width of 6 and a height of 1, and the second has a width of 3 and a height of 2, as shown.



- (a) Draw all the rectangles Ravi can form with 15 square tiles.
- (b) Draw all the rectangles Ravi can form with 24 square tiles.
- (c) Draw all the rectangles Ravi can form with 17 square tiles.
- (d) Can Ravi form more rectangles with 24 square tiles or 35 square tiles?
Justify your answer.
- (e) **Challenge:** Ravi has some number of tiles less than 100 and is able to form only one rectangle. What is the largest number of tiles that Ravi could have?





Problem of the Week

Problem A and Solution

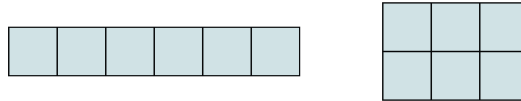
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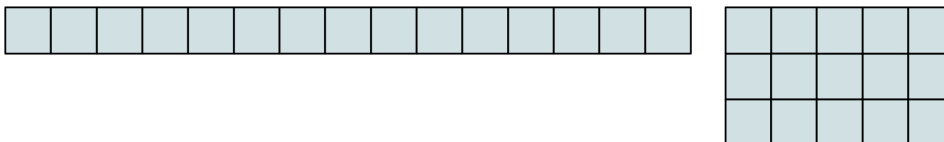
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- (d) Can Ravi form more rectangles with 24 square tiles or 35 square tiles?
Justify your answer.
- (e) **Challenge:** Ravi has some number of tiles less than 100 and is able to form only one rectangle. What is the largest number of tiles that Ravi could have?

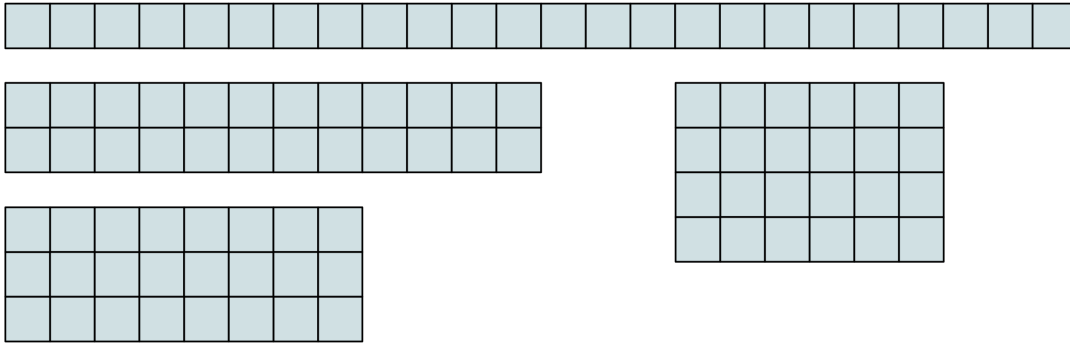
Solution

- (a) There are two possible rectangles that Ravi can form with 15 tiles.
They have dimensions 15×1 and 5×3 .





- (b) There are four possible rectangles that Ravi can form with 24 tiles. They have dimensions 24×1 , 12×2 , 8×3 , and 6×4 .



- (c) There is only one possible rectangle that Ravi can form with 17 tiles. It has dimensions 17×1 .



- (d) There are two possible rectangles that Ravi can form with 35 tiles. They have dimensions 35×1 and 7×5 . From part (b) we know that Ravi can form four rectangles with 24 tiles. So Ravi can form more rectangles with 24 square tiles than with 35 square tiles.

Another way to justify the answer is to notice the relationship between the number of rectangles Ravi can form and the number of whole numbers that are *divisors* of the number of tiles we start with in each case. Divisors are the whole numbers that divide exactly into the number of tiles that we start with. The whole number 1 is a divisor of every whole number. A whole number is always a divisor of itself.

There are 4 whole numbers that are divisors of 15: 1, 3, 5, 15, and Ravi can form 2 rectangles.

There are 8 whole numbers that are divisors of 24: 1, 2, 3, 4, 6, 8, 12, 24, and Ravi can form 4 rectangles.

There are 2 whole numbers that are divisors of 17: 1, 17, and Ravi can form 1 rectangle.

Generally, the more divisors a number has, the more rectangles Ravi can form with that number of tiles. Specifically, if there are an even number of divisors, Ravi can form half that number of rectangles. Note that if there are an odd number of divisors, then one of the rectangles Ravi can form will actually be a square. However, according to Ravi's rules, the width of each rectangle must be larger than its height, so squares are not allowed.

Since there are 4 whole numbers that are divisors of 35: 1, 5, 7, 35, then we predict that Ravi can form 2 rectangles. This means Ravi can form more rectangles with 24 tiles than with 35 tiles.

- (e) One way to figure out the largest number of tiles, less than 100, that can only form one rectangle is to look for the largest number that has only two whole numbers that are divisors of it: 1 and itself. Working backwards, we know that 99 is divisible by 3, and 98 is divisible by 2. However, 97 is not divisible by any whole numbers except 1 and 97. So the largest number of tiles that Ravi could have is 97.



Teacher's Notes

Rearranging identical square tiles into rectangles is a way to find the *factors* of a whole number. A factor of a whole number is another name for a divisor of a whole number.

A *prime number* is a whole number greater than 1 that has exactly two whole number factors: 1 and itself. So 2 is a prime number, but 4 is not a prime number. In our problem of finding the number of rectangles that can be formed by identical square tiles, a prime number of tiles will always result in one rectangle that has a height of 1 and a width equal to the number of tiles.

A *composite number* is a whole number greater than 1 that has more than two whole number factors. A composite number that is a perfect square has an odd number of whole number factors. Most factors come in pairs of two different numbers, where the two numbers multiplied together equal the number you are factoring. With a perfect square, one of the factors of the number is its square root. The square root of a number n is a number that when multiplied by itself is equal to n . So the factor that is equal to the square root does not have a different number to pair up with, which results in an odd number of factors.

In our problem, if we had a number of tiles that is a perfect square of a prime number, this would result in exactly one rectangle, since the width must be larger than its height. In these cases, we could form a square out of the tiles, where the length of the side of the square is equal to the square root of the number. An example of this would be the number 49. It has three whole number factors: 1, 7, and 49, and you can only form one rectangle (with dimensions 1×49) and one square (with dimensions 7×7) out of 49 tiles.



Problem of the Week

Problem A

Bean There; Done That

Suppose you want to make a four-bean salad using green beans, wax beans, kidney beans, and garbanzo beans. When you weigh the ingredients you notice the following:

- In total, the mass of all the beans in the salad is 1 kg.
- The sum of the mass of the garbanzo beans and the mass of the green beans make up half of the total mass of the beans in the salad.
- The masses of the garbanzo beans and the green beans are the same.
- The mass of the wax beans is 235 g.

What is the mass of each type of bean in the salad? Justify your answer.





Problem of the Week

Problem A and Solution

Bean There; Done That

Problem

Suppose you want to make a four-bean salad using green beans, wax beans, kidney beans, and garbanzo beans. When you weigh the ingredients you notice the following:

- In total, the mass of all the beans in the salad is 1 kg.
- The sum of the mass of the garbanzo beans and the mass of the green beans make up half of the total mass of the beans in the salad.
- The masses of the garbanzo beans and the green beans are the same.
- The mass of the wax beans is 235 g.

What is the mass of each type of bean in the salad? Justify your answer.

Solution

We know that 1 kg equals 1000 g. Half the total mass of the beans is $1000 \div 2 = 500$ g. So the total mass of the garbanzo beans and the green beans is 500 g. Since the remaining salad is made up of the other two kinds of beans, the total mass of the wax beans and the kidney beans must also be 500 g.

Since the masses of the garbanzo beans and the green beans are the same, then each mass is equal to $500 \div 2 = 250$ g.

Since the mass of the wax beans is 235 g, then the mass of the kidney beans must be $500 - 235 = 265$ g.

In summary, the salad contains:

- 250 g of garbanzo beans
- 250 g of green beans
- 235 g of wax beans
- 265 g of kidney beans



Problem of the Week

Problem A

Tri Kids Race

A triathlon is a race that has three components. Racers first complete a swimming component, then they complete a biking component, and finally they complete a running component.

In the Tri Kids race, the racers first swim 100 m. They then travel 25 m to the bicycle area. The racers then ride their bike in a 3 km long loop back to the bicycle area. The bicycle area is right beside the track. The racers then run 3 laps of the track to finish the race. One lap of the track is 400 m.

In the Tri Kids race, what is the total distance the racers have to cover, from start to finish?





Problem of the Week

Problem A and Solution

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In the Tri Kids race, what is the total distance the racers have to cover, from start to finish?

Solution

To calculate the total distance, we can convert all measurements to metres. The distance travelled on their bike is 3 km. Since 1 km is equal to 1000 m, then 3 km is equal to $3 \times 1000 = 3000$ m.

To calculate the distance the racers have to run, we multiply $3 \times 400 = 1200$ m.

We can add up the distances travelled in the swim, transition to the bicycle area, bike, and run. The total distance covered is: $100 + 25 + 3000 + 1200 = 4325$ m.

Alternatively, we can enumerate the distances travelled in a table:

Race Component	Distance of Component	Total Distance Travelled
Swim	100 m	100 m
Transition	25 m	125 m
Bike	3 km = 3000 m	3125 m
Lap 1	400 m	3525 m
Lap 2	400 m	3925 m
Lap 3	400 m	4325 m



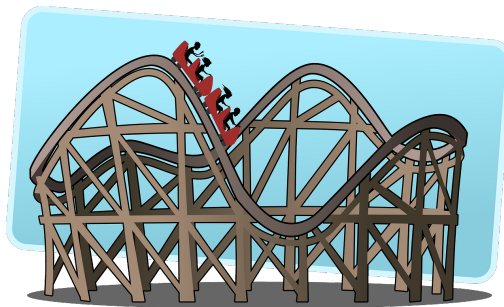
Problem of the Week

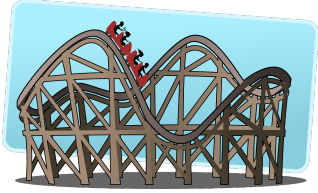
Problem A

Roller Coaster Riders

A local amusement park has many roller coasters. On the roller coaster Gargantuan, the train has 8 cars, seating 4 guests in each car at one time. A ride starts every 3 minutes.

If the roller coaster was full every time, how many people rode the Gargantuan in a half an hour from the first ride starting? Justify your answer.





Problem of the Week

Problem A and Solution

Roller Coaster Riders

Problem

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If the roller coaster was full every time, how many people rode the Gargantuan in a half an hour from the first ride starting? Justify your answer.

Solution

If there are 8 cars, and each car can hold 4 guests, then the maximum capacity of the roller coaster is $8 \times 4 = 32$. In other words, at most 32 people can ride the Gargantuan at one time.

Half an hour is equal to 30 minutes. Now we can make a table to keep track of the total number of riders, if 32 people ride the Gargantuan every 3 minutes.

Time Elapsed (minutes)	Total Number of Riders
3	32
6	64
9	96
12	128
15	160
18	192
21	224
24	256
27	288
30	320

Alternatively, we can calculate that in 30 minutes, there must be $30 \div 3 = 10$ rides completed. Since the maximum capacity of one ride is 32 people, then the maximum number of riders in 30 minutes is $32 \times 10 = 320$ people.



Problem of the Week

Problem A

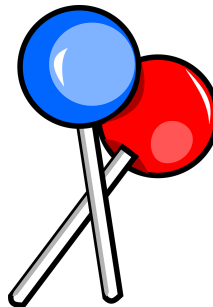
Sweet Treat

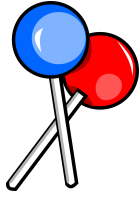
A single package of suckers contains 25 suckers of different colours: blue, red, yellow, pink, and green. There is an equal number of each colour of sucker in a package.

Miss Lolli surveys her class to find out what colour they would like to have for their special Fun Friday treat. The results of the survey are tallied as follows.

Colour	Number of Students
Blue	
Red	
Yellow	
Pink	
Green	

- (a) If Miss Lolli wants to make sure that each student receives the sucker colour they would like, how many packages of suckers does she need to buy?
- (b) After each student gets their sucker of choice on Friday, how many of each colour of sucker will she have left over?





Problem of the Week

Problem A and Solution

Sweet Treat

Problem

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- (b) After each student gets their sucker of choice on Friday, how many of each colour of sucker will she have left over?

Solution

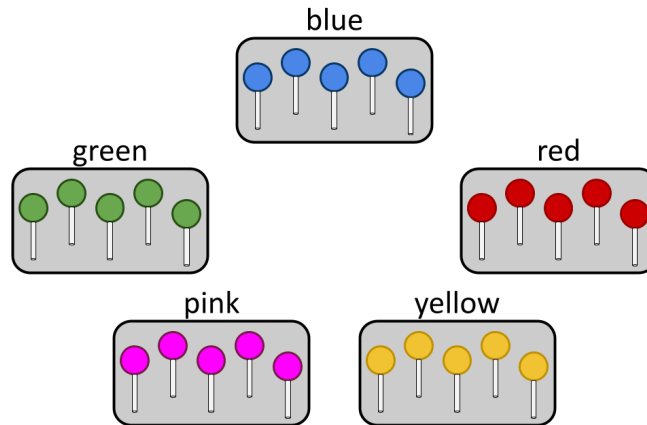
- (a) From the tally chart we know that the most popular colour is red, and 11 students selected that colour. This means Miss Lolli needs to buy enough packages to have at least 11 red suckers.

There are 25 suckers in a package, there are 5 different colours, and there are an equal number of each colour in a package. This means there are $25 \div 5 = 5$ suckers of each colour in a package.



Alternatively, we can find there are 5 suckers of each colour in a package in the following way:

If we build one pile for each colour, and we add the suckers from the package to a pile one at a time, we will find that there are 5 suckers of each colour.



Since there are 5 red suckers in a single package, then there are 10 red suckers in two packages, and 15 red suckers in three packages. This means Miss Lolli needs to buy three packages to get enough red suckers for the students.

- (b) Since there are 5 suckers of each colour in a package, then if Miss Lolli buys three packages there will be 15 suckers of each colour.

For each colour of sucker, we subtract the value of the tally from 15 to see how many will be left over of that colour:

- Blue: $15 - 4 = 11$ left over
- Red: $15 - 11 = 4$ left over
- Yellow: $15 - 2 = 13$ left over
- Pink: $15 - 7 = 8$ left over
- Green: $15 - 3 = 12$ left over



Problem of the Week

Problem A

Feeding the Pets

I give my dog 162 g of dog food each meal. He eats two meals per day.

I give my cat 95 g of cat food each meal. She eats one meal per day.

The dog food comes in a 5 kg bag. The cat food comes in a 2 kg bag.

If I start a new bag of food for each pet on the same day, which bag will run out first? Justify your answer.





Problem of the Week

Problem A and Solution

Feeding the Pets

Problem

I give my dog 162 g of dog food each meal. He eats two meals per day.

I give my cat 95 g of cat food each meal. She eats one meal per day.

The dog food comes in a 5 kg bag. The cat food comes in a 2 kg bag.

If I start a new bag of food for each pet on the same day, which bag will run out first? Justify your answer.

Solution

One way to determine which bag will run out first is to make a table that keeps track of how much food each pet consumes.

Since the dog eats two meals of 162 g each day, then he consumes $162 + 162 = 324$ g per day. Since $1 \text{ kg} = 1000 \text{ g}$, then $2 \text{ kg} = 2 \times 1000 = 2000 \text{ g}$ and $5 \text{ kg} = 5 \times 1000 = 5000 \text{ g}$. We will continue tracking the amount of food consumed until we reach one of these limits.

Day	Total Consumed by Cat (in g)	Total Consumed by Dog (in g)
1	95	324
2	190	648
3	285	972
4	380	1296
5	475	1620
6	570	1944
7	665	2268
8	760	2592
9	855	2916
10	950	3240
11	1045	3564
12	1140	3888
13	1235	4212
14	1330	4536
15	1425	4860
16	1520	5184

On Day 16 the dog will have consumed more than 5000 g of food, but the cat has not yet consumed 2000 g of food. So the dog food bag will run out first.

Alternatively, we could use estimation to determine which bag will run out first. Since 95 g is less than 100 g, then the cat food will last at least 20 days since $20 \times 100 = 2000$. Since 324 g is greater than 250 g, then the dog food will last less than 20 days since $20 \times 250 = 5000$.

Therefore, the cat food will last longer than the dog food, so the dog food bag will run out first.



Problem of the Week

Problem A

Bicycle Time

Mr. Turnblatt's bicycle needs to be serviced every 600 km. Each year, he rides his bike between March and November for 35 consecutive weeks, 5 days a week, 8 km a day.

- (a) If he has his bike serviced before his first trip of the year, how many weeks will it be until his bicycle needs the next service?
- (b) How many times will he have his bike serviced in one year?





Problem of the Week

Problem A and Solution

Bicycle Time

Problem

Mr. Turnblatt's bicycle needs to be serviced every 600 km. Each year, he rides his bike between March and November for 35 consecutive weeks, 5 days a week, 8 km a day.

- (a) If he has his bike serviced before his first trip of the year, how many weeks will it be until his bicycle needs the next service?
- (b) How many times will he have his bike serviced in one year?

Solution

- (a) Since Mr. Turnblatt rides his bike 5 days a week and 8 km a day, then in one week he travels $5 \times 8 = 40$ km.

We can skip count by 40 to see how many weeks it takes to get to 600 km:

40, 80, 120, 160, 200, 240, 280, 320, 360, 400, 440, 480, 520, 560, 600

So after 15 weeks Mr. Turnblatt will have ridden 600 km and will need to have his bike serviced.

Alternatively, we could have divided $600 \div 40 = 15$ to determine that it takes Mr. Turnblatt 15 weeks to ride 600 km.

- (b) From part (a), we know that his bicycle needs to be serviced every 15 weeks. So, the bike needs to be serviced at the end of Week 15 and at the end of Week 30. After that, since there are only 5 more weeks of biking, the bike does not need to be serviced until the following year.

Counting the service before the first trip of the year, Mr. Turnblatt will need to have his bike serviced 3 times in a year.



Problem of the Week

Problem A

Delivery Dilemma

At the beginning of the day, a delivery truck contains 368 packages.
The driver of the truck is making deliveries to various office buildings in a city.

Between 9 a.m. and 10 a.m., they deliver 66 packages.

Between 10 a.m. and 11 a.m., they deliver 103 packages.

Between 11 a.m. and 12 p.m., they deliver 88 packages.

When the driver returns to the depot for a lunch break, 273 packages are added to the truck.

Between 1 p.m. and 2 p.m., they deliver 111 packages.

Between 2 p.m. and 3 p.m., they deliver 86 packages.

At 4 p.m., they have 99 packages left in the truck. How many packages did they deliver between 3 p.m. and 4 p.m.?





Problem of the Week

Problem A and Solution

Delivery Dilemma

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At 4 p.m., they have 99 packages left in the truck. How many packages did they deliver between 3 p.m. and 4 p.m.?

Solution

Since they start with 368 packages in the truck, at 10 a.m. they will have
 $368 - 66 = 302$ packages in the truck.

Then at 11 a.m. they will have $302 - 103 = 199$ packages in the truck, and at 12 p.m. they will have $199 - 88 = 111$ packages in the truck.

At lunch they add 273 packages to the truck, so there are now $111 + 273 = 384$ packages in the truck.

Thus, in the afternoon, they start with 384 packages in the truck.

Then at 2 p.m. they will have $384 - 111 = 273$ packages in the truck, and at 3 p.m. they will have $273 - 86 = 187$ packages in the truck.

Since there are 99 packages on the truck at 4 p.m., then there must have been
 $187 - 99 = 88$ packages delivered between 3 p.m. and 4 p.m.



Problem of the Week

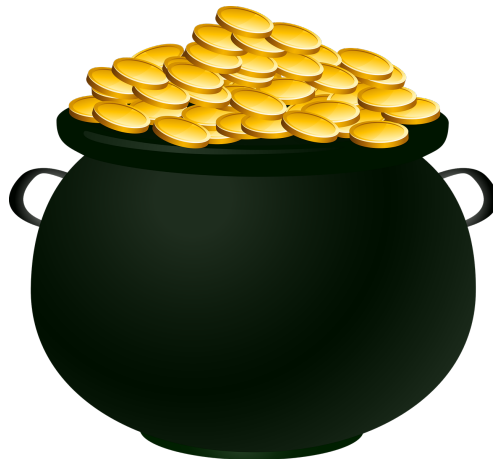
Problem A

Patterned Savings

Rebecca begins saving money starting on January 1. She collects money in a jar in the following way. Every day she puts a quarter in the jar. Every second day, starting on January 2, she puts a loonie in the jar. Every fifth day, starting on January 5, she puts a toonie in the jar.

So some days she puts one coin in the jar, some days she puts two coins in the jar, and some days she puts three coins in the jar. In Canada, a quarter is worth 25 cents, a loonie is worth one dollar, and a toonie is worth two dollars. There are 100 cents in one dollar.

- (a) How many coins does she add to the jar on January 12?
- (b) How many coins does she add to the jar on January 23?
- (c) How many coins does she add to the jar on January 30?
- (d) How many coins in total does she have in the jar by the end of January?
- (e) If she keeps saving this way, how much money will she have after 90 days?





Problem of the Week

Problem A and Solution

Patterned Savings

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- How many coins does she add to the jar on January 12?
- How many coins does she add to the jar on January 23?
- How many coins does she add to the jar on January 30?
- How many coins in total does she have in the jar by the end of January?
- If she keeps saving this way, how much money will she have after 90 days?

Solution

One way to answer most of these questions is to keep track of how many coins are added each day and how much money is accumulated each day in a table.

Day	Number of Quarters Added	Number of Loonies Added	Number of Toonies Added	Money Added	Total Coins	Total Money
1	1	0	0	\$0.25	1	\$0.25
2	1	1	0	\$1.25	3	\$1.50
3	1	0	0	\$0.25	4	\$1.75
4	1	1	0	\$1.25	6	\$3.00
5	1	0	1	\$2.25	8	\$5.25
6	1	1	0	\$1.25	10	\$6.50
7	1	0	0	\$0.25	11	\$6.75
8	1	1	0	\$1.25	13	\$8.00
9	1	0	0	\$0.25	14	\$8.25
10	1	1	1	\$3.25	17	\$11.50

We could continue the table, and we would see that the amounts of coins and money added would repeat every 10 days. We observe that on every day that is a



multiple of 2 (except on day 10) Rebecca adds two coins to the jar, and on every day that is a multiple of 5 (except on day 10) Rebecca adds two coins to the jar. On day 10, which is a multiple of both 2 and 5, Rebecca adds three coins to the jar. On all other days, Rebecca adds just one coin to the jar.

- (a) Since 12 is a multiple of 2 but not 5, on January 12, Rebecca adds 2 coins to the jar.
- (b) Since 23 is neither a multiple of 2 nor a multiple of 5, Rebecca adds only 1 coin to the jar.
- (c) Since 30 is a multiple of both 2 and 5, Rebecca adds 3 coins to the jar.
- (d) From the table, we know that after 10 days, Rebecca will have saved 17 coins. This pattern will repeat every 10 days. So by January 30, the pattern will have repeated 3 times. On January 31, she will add one more quarter. So, by January 31 Rebecca will have $17 + 17 + 17 + 1 = 52$ coins in the jar.
- (e) Every 10 days Rebecca will have saved a total of \$11.50. This pattern will repeat 9 times over a 90 day period. So after 90 days Rebecca will have $\$11.50 + \$11.50 + \$11.50 + \$11.50 + \$11.50 + \$11.50 + \$11.50 + \$11.50 + \$11.50 = \103.50 .

Alternatively we could calculate the savings as $9 \times \$11.50 = \103.50 .



Teacher's Notes

This problem shows a pattern that repeats every 10 days. In mathematics, we could refer to this kind of repetition as *periodic*. The length of the interval between repeated elements is known as the *period* of the function. In this case, the period of the savings function is 10, which is the *least common multiple* or *LCM* of the integers 1, 2, and 5. The LCM of a set of integers is the smallest positive integer that is a multiple of each integer in the set. In this case, we are looking for the smallest positive multiple of each of the individual periods of savings.

Periodic functions appear in mathematics and in the real world. Trigonometric functions such as *sin*, *cos*, and *tan* are periodic functions. Sound waves, phases of the moon, and your blood pressure, are all examples of periodic functions in nature.



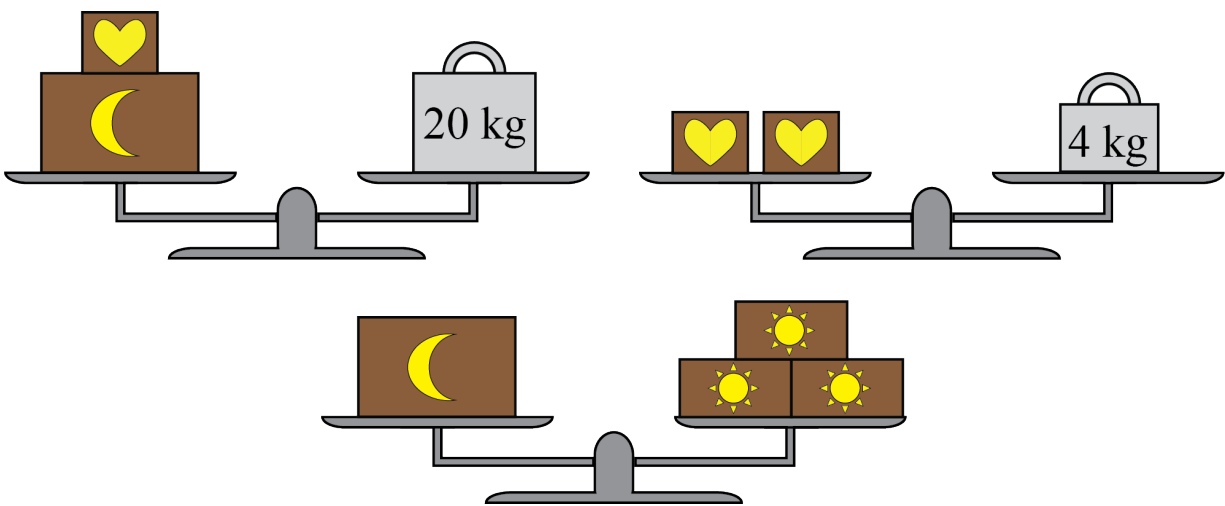
Problem of the Week

Problem A

Balancing Act

Robbie is in charge of sending out boxes from a distribution centre. The contents of the boxes are identified by shapes stamped on them: a heart, a moon, or a sun. All boxes with the same stamp have the same mass.

The following diagrams show what Robbie observed when arranging some of the boxes and standard weights on a scale.



Given that each scale is balanced, determine the mass of each box.



Problem of the Week

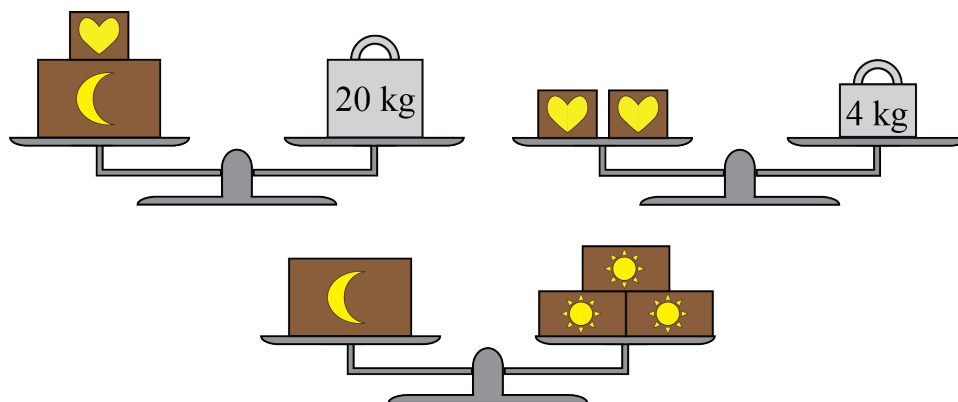
Problem A and Solution

Balancing Act

Problem

Robbie is in charge of sending out boxes from a distribution centre. The contents of the boxes are identified by shapes stamped on them: a heart, a moon, or a sun. All boxes with the same stamp have the same mass.

The following diagrams show what Robbie observed when arranging some of the boxes and standard weights on a scale.



Given that each scale is balanced, determine the mass of each box.

Solution

From the diagrams we notice the following.

- One moon box and one heart box have a total mass of 20 kg.
- Two heart boxes have a mass of 4 kg.
- One moon box has the same total mass as three sun boxes.

Since two heart boxes have a mass of 4 kg, then the mass of one heart box is $\frac{1}{2}$ of 4 kg. Therefore, one heart box has a mass of 2 kg.

Since one heart box and one moon box have a total mass of 20 kg, and one heart box has a mass of 2 kg, then one moon box has a mass of $20 - 2 = 18$ kg.

Since 3 sun boxes have the same total mass as one moon box, then 1 sun box must be $\frac{1}{3}$ the mass of one moon box. Since $\frac{1}{3}$ of 18 is 6, one sun box must have a mass of 6 kg.

Therefore, one heart box has a mass of 2 kg, one moon box has a mass of 18 kg, and one sun box has a mass of 6 kg.



Teacher's Notes

The idea of a balance scale is a nice analogy for an algebraic equation. We can represent the information in the problem using equations with variables to represent the masses of the different types of boxes. Here is one way to solve the problem algebraically.

Let x represent the mass of a heart box, in kg.

Let y represent the mass of a sun box, in kg.

Let z represent the mass of a moon box, in kg.

From the information in the diagrams, we can write the following equations:

$$x + z = 20 \quad (1)$$

$$2x = 4 \quad (2)$$

$$z = 3y \quad (3)$$

We can divide both sides of equation (2) by 2 to get

$$\begin{aligned} \frac{2x}{2} &= \frac{4}{2} \\ x &= 2 \end{aligned}$$

Now, substituting $x = 2$ into equation (1), we get

$$2 + z = 20$$

Subtracting 2 from each side of this equation, we get

$$z = 18$$

Finally, substituting $z = 18$ into equation (3), we get

$$18 = 3y$$

Dividing both sides of this equation by 3, we get

$$\begin{aligned} \frac{18}{3} &= \frac{3y}{3} \\ 6 &= y \end{aligned}$$

Therefore, one heart box has a mass of 2 kg, one moon box has a mass of 18 kg, and one sun box has a mass of 6 kg.



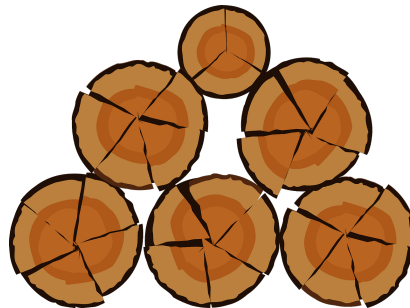
Problem of the Week

Problem A

Piling Wood

The Taylor family heats their home with wood. They recently got a large delivery of cut logs which were left in a pile on their driveway. Janelle and Alphonso carry logs from the pile and stack them in the woodshed. In one trip, Janelle can carry 4 logs at a time and Alphonso can carry 3 logs at a time. They each take the same amount of time to carry logs from the pile, stack them in the woodshed, and walk back to the pile.

- (a) If they each take as many logs as they can on each trip from the pile to the woodshed, how many logs have Janelle and Alphonso stacked after making 10 trips from the pile to the woodshed?
- (b) When Janelle and Alphonso started, there were 200 logs in the pile. How many trips from the pile to the woodshed do they have to take in order to move all the logs?
- (c) It takes approximately 30 seconds for Janelle and Alphonso to carry logs from the pile, stack them in the woodshed, and return to the pile. Approximately how many minutes does it take them to stack all of the logs in part (b)?
- (d) On the last trip, Janelle and Alphonso carry the same number of logs. If they each take as many logs as they can on their earlier trips, how many logs do they each carry on the last trip?





Problem of the Week

Problem A and Solution

Piling Wood

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- If they each take as many logs as they can on each trip from the pile to the woodshed, how many logs have Janelle and Alphonso stacked after making 10 trips from the pile to the woodshed?
- When Janelle and Alphonso started, there were 200 logs in the pile. How many trips from the pile to the woodshed do they have to take in order to move all the logs?
- It takes approximately 30 seconds for Janelle and Alphonso to carry logs from the pile, stack them in the woodshed, and return to the pile. Approximately how many minutes does it take them to stack all of the logs in part (b)?
- On the last trip, Janelle and Alphonso carry the same number of logs. If they each take as many logs as they can on their earlier trips, how many logs do they each carry on the last trip?

Solution

- In total, Janelle and Alphonso carry $4 + 3 = 7$ logs per trip from the log pile to the woodshed. This means they carry $7 \times 10 = 70$ logs in 10 trips.
- To figure out how many trips it takes to move all the logs, we could skip count by 7s until we get to or pass 200:

7, 14, 21, 28, 35, 42, 49, 56, 63, 70, 77, 84, 91, ...

Then we would count how many 7s it takes to get past 200 and we would see that it is 29.

Alternatively, we know from part (a) that after 10 trips, they will have moved 70 logs. So, after 20 trips they will have moved 140 logs, and after 30 trips they will have moved 210 logs. If we count backwards from 210, we can



determine that after 29 trips they will have moved 203 logs, and after 28 trips they will have moved 196 logs. Therefore, we can conclude that it takes 29 trips to move all the logs.

Another way to calculate this is to divide 200 by 7 to obtain 28 remainder 4. Since there is a remainder after 28 trips, then it will take one more trip to move all the logs. Therefore, we can conclude that it takes 29 trips to move all the logs.

- (c) Since each trip takes approximately 30 seconds, then 2 trips take approximately 1 minute. This means that 28 trips would take approximately 14 minutes. Therefore, 29 trips would take approximately 14 and a half minutes.
- (d) After 28 trips, they have moved $28 \times 7 = 196$ logs. This means there are $200 - 196 = 4$ logs left for their last trip. Since Janelle and Alphonso carry the same number of logs on their last trip, then they each must carry 2 logs.



Problem of the Week

Problem A

What's in the Pouch?

Zoha's class is raising money for a local charity. The class puts any money raised into a pouch, and each Thursday their teacher creates a math problem about the money in the pouch.

The following note was attached to the pouch today.

This pouch contains a total of \$20.30 in Canadian money consisting of 4 coins and 3 bills.
What are the specific bills and coins in the pouch?

What is the solution to the problem? Justify your answer.

Note: The coins available in Canada are nickels that are worth 5 cents, dimes that are worth 10 cents, quarters that are worth 25 cents, loonies that are worth \$1, and toonies that are worth \$2. Also, \$1 is equal to 100 cents. The lowest denominations of bills are worth \$5, \$10, and \$20.





Problem of the Week

Problem A and Solution

What's in the Pouch?

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Solution

The pouch cannot include a \$20 bill since there is only 30 cents more than \$20, and that would mean the pouch only contained 1 bill. Similarly, it cannot include two \$10 bills since this would mean the pouch only contained 2 bills.

If it has one \$10 bill and two \$5 bills, then that would be a total of \$20. This is three bills. In this case, there are 30 cents remaining, which can be formed by:

- 1 quarter and 1 nickel for a total of 2 coins
- 3 dimes for a total of 3 coins
- 2 dimes and 2 nickels for a total of 4 coins
- 1 dime and 4 nickels for a total of 5 coins
- 6 nickels for a total of 6 coins

So one possibility is that the pouch contains one \$10 bill, two \$5 bills, two dimes, and two nickels. However, we should check to see if this is the only possibility.

Could it have three \$5 bills which is \$15? This means there would be \$5.30 remaining. The fewest number of coins you need to make \$5 is two toonies and one loonie, which is a total of 3 coins. But you need at least 2 coins to make up 30 cents. So you need at least 5 coins to make \$5.30, which is too many coins.

Any more attempts to come up with \$20 would take more bills and coins. So the only possibility that meets the requirements of the problem is one \$10 bill, two \$5 bills, two dimes, and two nickels.



Problem of the Week

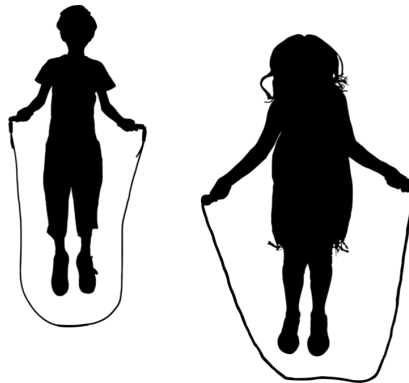
Problem A

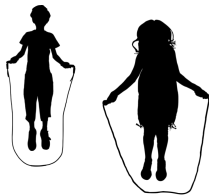
Skipping at School

Sophia is an excellent skipper and skips every morning recess with her friends at school. At the end of each recess, she counts how many times she has jumped.

Sophia rarely misses her jumps or gets caught on the skipping rope, and so she averages 60 jumps every minute.

- (a) If recess is 10 minutes long, how many jumps do you expect Sophia to complete in a single recess?
- (b) How many days will it take for Sophia to complete at least 5000 jumps, if she jumps for one recess every day?





Problem of the Week

Problem A and Solution

Skipping at School

Problem

Sophia is an excellent skipper and skips every morning recess with her friends at school. At the end of each recess, she counts how many times she has jumped.

Sophia rarely misses her jumps or gets caught on the skipping rope, and so she averages 60 jumps every minute.

- If recess is 10 minutes long, how many jumps do you expect Sophia to complete in a single recess?
- How many days will it take for Sophia to complete at least 5000 jumps, if she jumps for one recess every day?

Solution

- Since Sophia jumps an average of 60 times in 1 minute, and recess is 10 minutes long, we can expect Sophia to complete $60 \times 10 = 600$ jumps in a single recess.
- From part (a) we expect Sofia to complete 600 jumps in one recess. Since she jumps for one recess every day, then in two days, we would expect her to complete $600 + 600 = 1200$ jumps. We can make a table to determine how long it will take Sophia to complete at least 5000 jumps.

Days	Total Jumps
1	600
2	$600 + 600 = 1200$
3	$1200 + 600 = 1800$
4	$1800 + 600 = 2400$
5	$2400 + 600 = 3000$
6	$3000 + 600 = 3600$
7	$3600 + 600 = 4200$
8	$4200 + 600 = 4800$
9	$4800 + 600 = 5400$

After 8 days, we would expect Sophia to have completed 4800 jumps, and after 9 days we would expect Sophia to have completed 5400 jumps. Therefore, it will take Sophia 9 days to complete at least 5000 jumps.



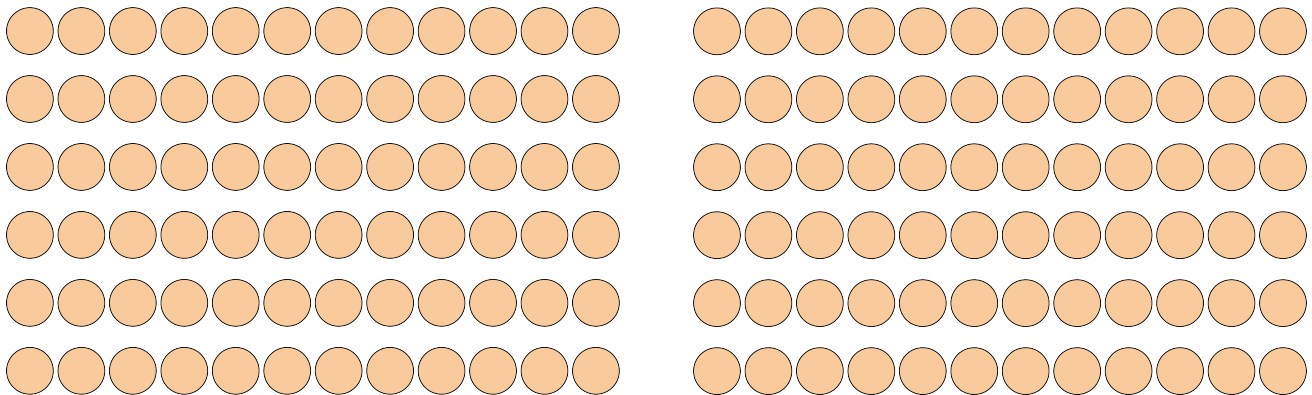
Problem of the Week

Problem A

Graduation Ceremony

At Emily Carr Elementary School, the Grade 8 students are graduating and are having a ceremony. Tickets are given to the graduates and the school staff for free. Parents and other students need to buy tickets.

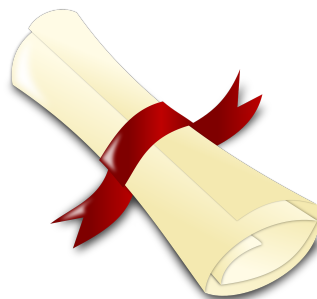
Staff set up 6 rows of 24 chairs in the gym. The diagram below shows the arrangement.



We know that

- $\frac{1}{3}$ of the chairs are filled with adults (parents/relatives) who bought tickets for \$4 each,
- $\frac{1}{4}$ of the chairs are filled with other students who bought tickets for \$2 each,
- $\frac{1}{6}$ of the chairs are filled with staff of the school, and
- the remaining chairs are filled with the graduates.

- (a) How many graduates are at the ceremony?
- (b) How much money was collected from ticket sales?





Problem of the Week

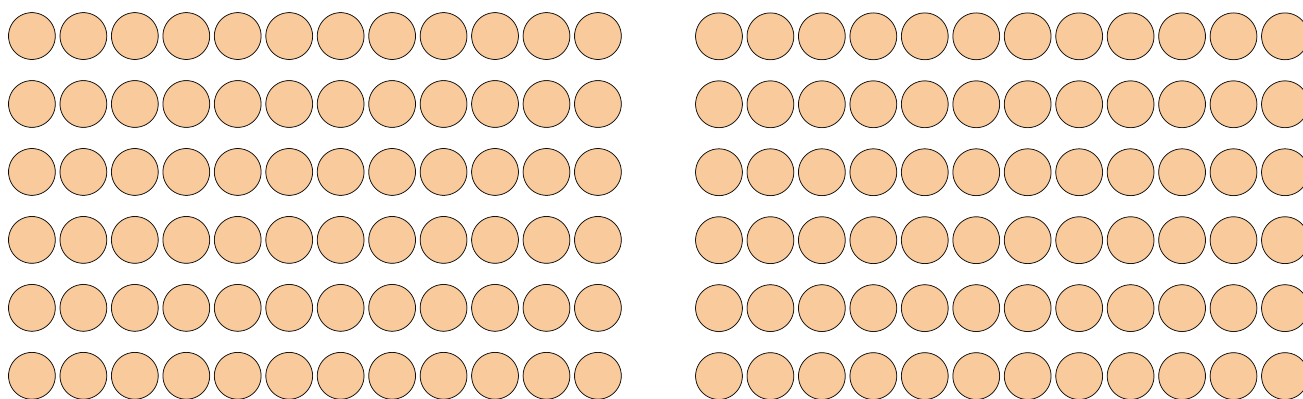
Problem A and Solution

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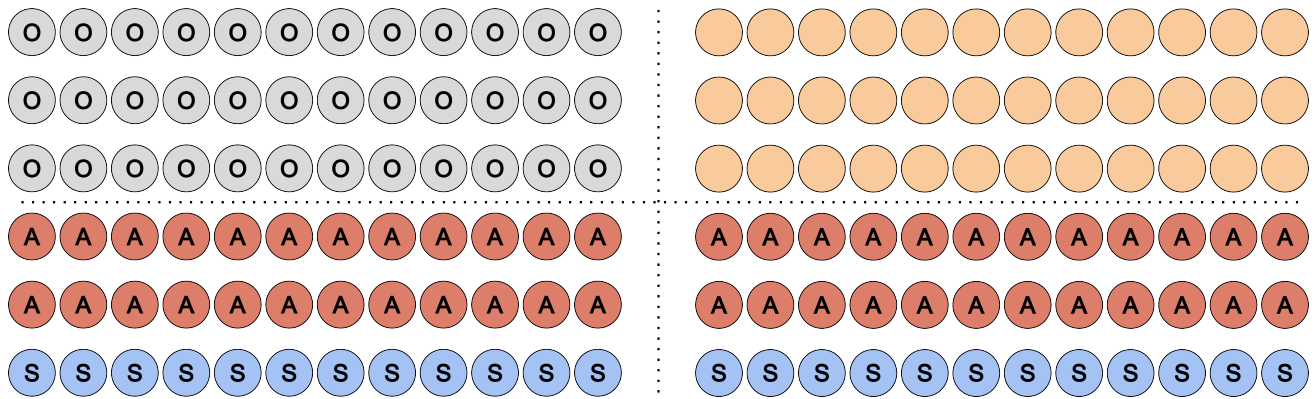
Solution

We will first figure out how many of each category of people are at the graduation. One way to do this is to use the diagram showing the setup of the chairs.

Since there are 6 rows of chairs, each row contains $\frac{1}{6}$ of the total number of chairs. Since $\frac{1}{6}$ of the chairs are filled with the staff of the school, the number of chairs filled with staff is the same as the number of chairs in one row. We illustrate this by marking all of the chairs in one row (the sixth row in the diagram) with the letter *S*.

We can use a line down the centre aisle and a line between the third and fourth rows to divide the setup of the chairs into 4 equal parts. Since $\frac{1}{4}$ of the chairs are filled with other students, we mark all of the chairs in one of the quarters (the top-left quarter in the diagram) with the letter *O*. Of course, we chose a quarter that did not already have marked chairs.

Also, if we divide the number of rows into 3 equal parts, that would result in two rows in each part. So two rows contain $\frac{1}{3}$ of the total number of chairs. Since $\frac{1}{3}$ of the chairs are filled with adults, we mark all of the chairs in two rows (the fourth and fifth rows in the diagram) with the letter *A*.



Counting the number of chairs in each category, we get:

24 staff, 48 adults, and 36 other students.

(a) Counting the number of unmarked chairs, we get 36 chairs remaining that are filled with graduates.

(b) Since there are 48 adults who paid \$4 per ticket, the total amount paid by adults is $48 \times \$4 = \192 .

Since there are 36 other students who paid \$2 per ticket, the total amount paid by other students is $36 \times \$2 = \72 .

The total amount collected from ticket sales is $\$192 + \$72 = \$264$.

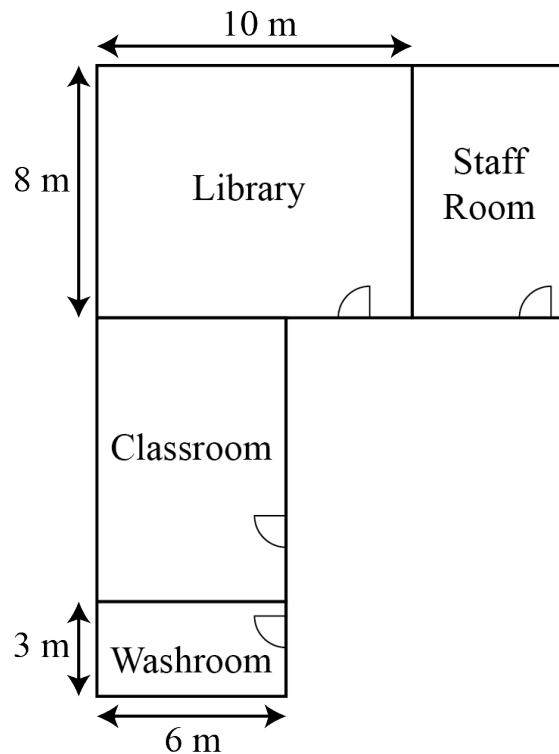


Problem of the Week

Problem A

Room to Grow

Ada Lovelace Public School needs to add a new wing to its main building that includes a classroom, library, staff room, and washroom. The plans for the addition are shown, with all new rooms being rectangular; however, some of the measurements are missing.



Fortunately, the contractor remembered the following extra details:

The length of the classroom is three times the width of the washroom.
The width of the staff room is half the length of the library.

When describing the dimensions of a rectangle, the contractor uses *length* to describe the longer side, and *width* to describe the shorter side.

- What are the missing dimensions of the classroom and the staff room?
- What is the total floor area of the four new rooms?



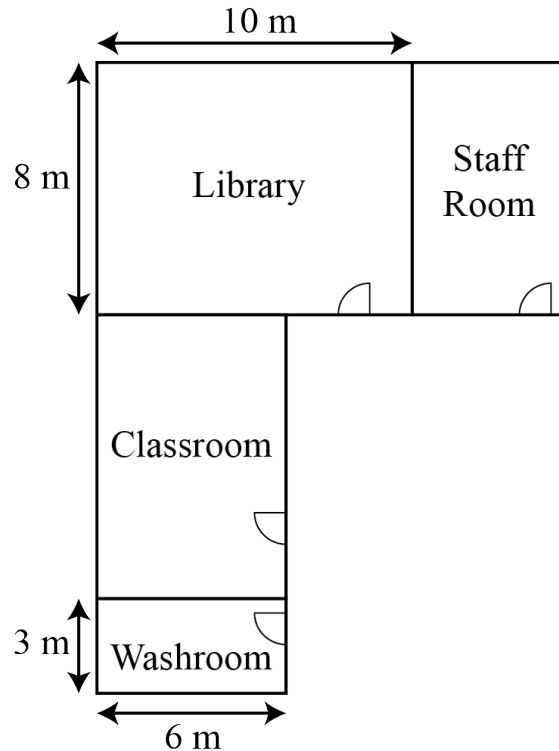
Problem of the Week

Problem A and Solution

Room to Grow

Problem

Ada Lovelace Public School needs to add a new wing to its main building that includes a classroom, library, staff room, and washroom. The plans for the addition are shown, with all new rooms being rectangular; however, some of the measurements are missing.



Fortunately, the contractor remembered the following extra details:

The length of the classroom is three times the width of the washroom.
The width of the staff room is half the length of the library.

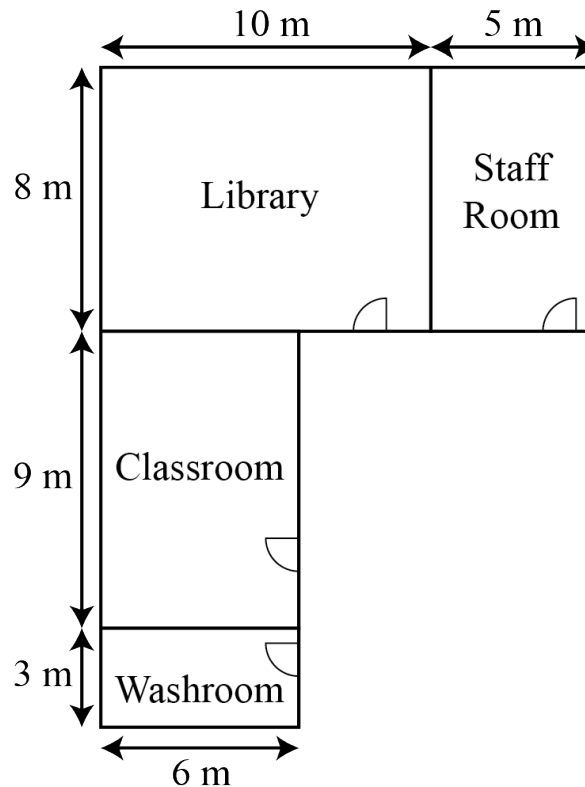
When describing the dimensions of a rectangle, the contractor uses *length* to describe the longer side, and *width* to describe the shorter side.

- What are the missing dimensions of the classroom and the staff room?
- What is the total floor area of the four new rooms?



Solution

- (a) Since the length of the classroom is three times the width of the washroom, its length is $3 \times 3 = 9$ m. Since the width of the staff room is half the length of the library, its width is $10 \div 2 = 5$ m. These dimensions are shown in the diagram.



- (b) There are many ways to determine the total floor area.

Solution 1: We calculate the area of each room and add those areas together.

- The area of the washroom is $6 \times 3 = 18$ m².
- The area of the classroom is $9 \times 6 = 54$ m².
- The area of the library is $8 \times 10 = 80$ m².
- The area of the staff room is $8 \times 5 = 40$ m².

Therefore, the total floor area is $18 + 54 + 80 + 40 = 192$ m².

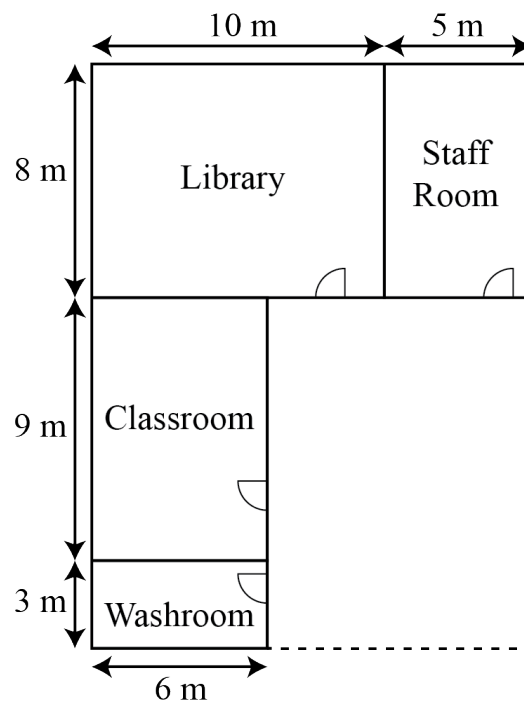
Solution 2: We calculate the area of the classroom based on the area of the washroom, and calculate the area of the staff room based on the area of the library. Since the length of the classroom is three times the width of the washroom, but the width of the classroom is the same as the length of the



washroom, the area of the classroom is three times the area of the washroom. We can imagine that we could put three washrooms side-by-side and they would fill in the same area as the classroom. Since the area of the washroom is $6 \times 3 = 18 \text{ m}^2$, it follows that the area of the classroom is $18 \times 3 = 54 \text{ m}^2$. Similarly, the area of the staff room would fill half of the area of the library. Since the area of the library is $8 \times 10 = 80 \text{ m}^2$, it follows that the area of the staff room is $80 \div 2 = 40 \text{ m}^2$.

Therefore, the total floor area is $18 + 54 + 80 + 40 = 192 \text{ m}^2$.

Solution 3: We square off the diagram to create a large rectangle, as shown.



This large rectangle measures $10 + 5 = 15 \text{ m}$ across the top and $8 + 9 + 3 = 20 \text{ m}$ along the side. The total area of this rectangle is then $15 \times 20 = 300 \text{ m}^2$.

However, there is a section of the large rectangle that is not part of the floor plan, and hence its area needs to be subtracted. That section is a rectangle with length $3 + 9 = 12 \text{ m}$ and width $15 - 6 = 9 \text{ m}$. The area of the section is then $12 \times 9 = 108 \text{ m}^2$.

Therefore, the total floor area is equal to $300 - 108 = 192 \text{ m}^2$.



Problem of the Week

Problem A

Babysitting Bonus

Anya agrees to look after her younger brother every day for 1 hour before dinner. Her parents agree to pay her \$15 per week, starting in September. If she does a spectacular job, her parents agree that on the first Monday of each month she will get a raise of \$2 per week. So far, Anya has done a spectacular job.

Anya's parents have a monthly household budget. In the budget, they estimate how much they will be spending on different things each month. Determine the first month when her parents should estimate spending more than \$100 per month for paying Anya to babysit. Justify your answer.





Problem of the Week

Problem A and Solution

Babysitting Bonus

Problem

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Anya's parents have a monthly household budget. In the budget, they estimate how much they will be spending on different things each month. Determine the first month when her parents should estimate spending more than \$100 per month for paying Anya to babysit. Justify your answer.

Solution

We will estimate how much Anya earns in each month by approximating that there are 4 weeks in one month. The estimated monthly earnings are summarized in the table below.

Month	Earnings per Week	Estimated Earnings per Month
September	\$15	$4 \times \$15 = \60
October	$\$15 + \$2 = \$17$	$4 \times \$17 = \68
November	$\$17 + \$2 = \$19$	$4 \times \$19 = \76
December	$\$19 + \$2 = \$21$	$4 \times \$21 = \84
January	$\$21 + \$2 = \$23$	$4 \times \$23 = \92
February	$\$23 + \$2 = \$25$	$4 \times \$25 = \100
March	$\$25 + \$2 = \$27$	$4 \times \$27 = \108

Using these estimations, Anya's parents should budget more than \$100 per month starting in March.

NOTE: There is a possibility that either December or January will have five Mondays. (During a leap year, there could possibly be five Mondays in February.) In this case, Anya's parents would be paying her more than \$100 for babysitting during that month. However, the budget is an estimation of expenses, so it is reasonable for them to start budgeting more than \$100 for Anya's babysitting starting in March.



Problem of the Week

Problem A

Scrabble Words

Scrabble™ is a game where players place tiles containing individual letters on a board to form words. The board is divided up into squares, and each letter in a word is placed in adjacent squares in a row (reading from left to right) or a column (reading from the top down). Each letter has a point value, and a player's score for placing a word on the board is the sum of the point values for the letters in the word. The point values for each letter are in the following table.

Letter	A	B	C	D	E	F	G	H	I	J	K	L	M
Value	1	3	3	2	1	4	2	4	1	8	5	1	3
Letter	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
Value	1	1	3	10	1	1	1	1	4	4	8	4	10

In the Scrabble board shown, the coloured squares have special values.

Blue squares, marked **B** on the board, are double letter squares, which means if you put a tile on that square, then the letter is worth twice as many points.

Grey squares, marked **G** on the board, are triple letter squares, which means if you put a tile on that square, then the letter is worth three times as many points.

Red squares, marked **R** on the board, are double word squares, which means if you put a tile on that square, then the whole word is worth twice as many points.

Orange squares, marked **O** on the board, are triple word squares, which means if you put a tile on that square, then the whole word is worth three times as many points.

Double and triple letter scores are calculated before the double and triple word scores.

G				G	
	R				
		R			B
			R		
G				R	
		B			O

- How many points would the word EXACT be worth if you put the first letter in the first row and second column, and place the word horizontally across the board?
- How many points would the word DEBUT be worth if you put the first letter in the second row and second column, and place the word vertically on the board?
- How many points would the word CHEF be worth if you put the first letter of the word in the last column and third row, and place the word vertically on the board?



Problem of the Week

Problem A and Solution

Scrabble Words

Problem

Scrabble™ is a game where players place tiles containing individual letters on a board to form words. The board is divided up into squares, and each letter in a word is placed in adjacent squares in a row (reading from left to right) or a column (reading from the top down). Each letter has a point value, and a player’s score for placing a word on the board is the sum of the point values for the letters in the word. The point values for each letter are in the following table.

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Value	1	1	3	10	1	1	1	1	4	4	8	4	10

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- How many points would the word CHEF be worth if you put the first letter of the word in the last column and third row, and place the word vertically on the board?



Solution

- (a) If we place the letters for EXACT horizontally on the board starting with the letter E in the first row and second column, then the letter C will be on a grey square. This means that letter is worth three times its normal value. That is, the letter C is worth $3 \times 3 = 9$ points. So the score for this word is $1 + 8 + 1 + 9 + 1 = 20$ points.
- (b) If we place the letters for DEBUT vertically down the board starting with the D in the second row and second column, then the word covers a red square. This means the word is worth twice its normal value. The normal score for the word would be $2 + 1 + 3 + 1 + 1 = 8$. The red square means that the word would be worth twice as much, so the score for the word at this location on the board is $2 \times 8 = 16$ points.
- (c) If we place the letters for CHEF vertically down the board starting with the letter C in the last column and third row, then the letter C will be on a blue square. This means that letter is worth twice its normal value. That is, in this placement of the word, the letter C is worth $2 \times 3 = 6$ points. The word also covers an orange square, which means the word is worth three times its normal value. We calculate the score including the double value of the letter C before applying the triple word score. The score for the word without considering the triple word square, but including its double letter score, is $6 + 4 + 1 + 4 = 15$. When we consider the triple word square, the result is $3 \times 15 = 45$ points.



Teacher's Notes

When we apply the rule that double and triple letter scores are calculated before the double and triple word scores, we are enforcing an *order of operations* to our calculations.

We often use the mnemonic **BEDMAS** to describe the standard order of operations for mathematical operators. This stands for **B**rackets, **E**xponents, **D**ivision and **M**ultiplication in the order that they appear, and **A**ddition and **S**ubtraction in the order that they appear.

We use brackets in places where we want some operation to take precedence over another operation that would otherwise happen first according to BEDMAS. For example, in part (b) of this problem, we want to add the letter values together before we apply the double word score. We could calculate the result like this:

$$(2 + 1 + 3 + 1 + 1) \times 2 = (8) \times 2 = 16$$

Without the brackets the calculation

$$2 + 1 + 3 + 1 + 1 \times 2$$

would equal

$$2 + 1 + 3 + 1 + 2 = 9$$

since according to BEDMAS the multiplication is done before the addition.

In part (c), we need to multiply the point value of the letter C by 2 since it is on a double letter square. Then we add the rest of the letter values together before multiplying that sum by 3. So we could calculate the result for part (c) like this:

$$(2 \times 3 + 4 + 1 + 4) \times 3 = (6 + 4 + 1 + 4) \times 3 = (15) \times 3 = 45$$

Note that we do not need brackets around 2×3 since, according to BEDMAS, the multiplication will be done before the addition in the calculation. However, sometimes we use brackets (even when they are not required) to make calculations clearer. To emphasize that the double letter score must be calculated before the sum of the letters is calculated, it may be easier to understand the calculation for part (c) if we used this expression:

$$((2 \times 3) + 4 + 1 + 4) \times 3$$