# Problem of the Week
Problem A and Solution
Balancing Act

Problem

Bailey is in charge of sending out boxes from a distribution centre. The contents of the boxes are identified by shapes stamped on them: a heart, a moon, or a sun. All boxes with the same stamp have the same mass, and the cost of sending a box depends on its mass. Bailey has a balance scale and a few standard weights to help with the job. The following diagrams show what Bailey observed when arranging some of the boxes and standard weights on the scales.
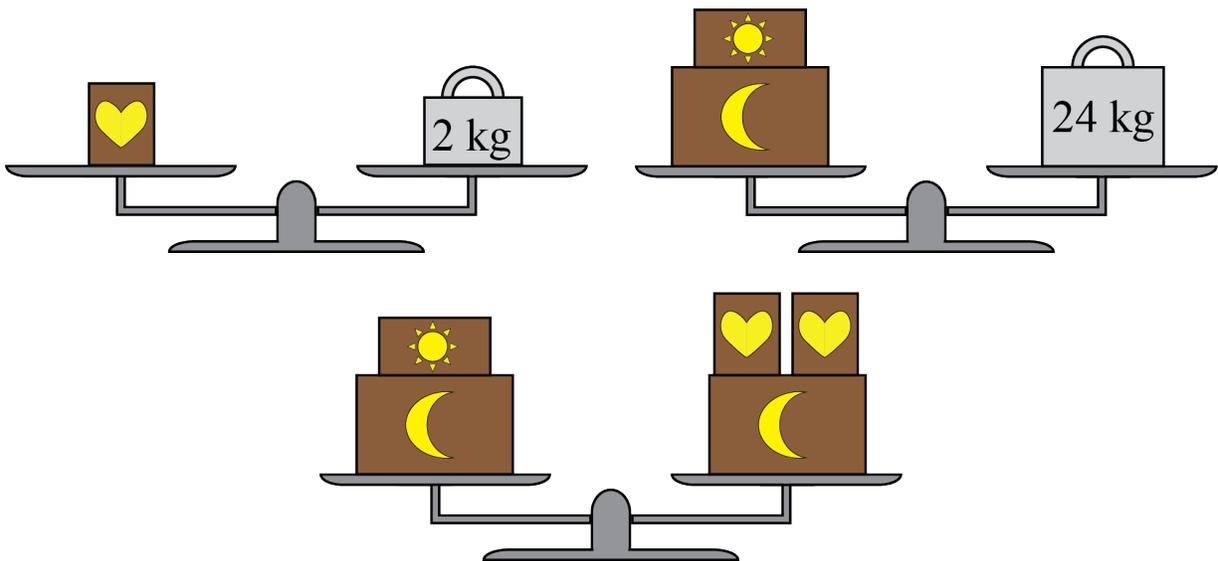
Find the mass of each box.

Solution

From the diagrams we notice the following.

- One heart box has a mass of 2 kg.

- One moon box and one sun box have a total mass of 24 kg.

- One moon box and one sun box have the same total mass as one moon box and two heart boxes.

From this, we can conclude that one moon box and two heart boxes have a total mass of 24 kg. Also, two heart boxes have the same mass as one sun box.

Since one heart box has a mass of 2 kg, then two heart boxes have a mass of 4 kg. Therefore, one sun box has a mass of 4 kg.

This means 4 kg + (mass of a moon box) = 24 kg. Since $4 + 20 = 24$, we can determine that one moon box must have a mass of 20 kg.

Teacher's Notes

The idea of a balance scale is a nice analogy for an algebraic equation. We can represent the information in the problem using equations with variables to represent the masses of the different types of boxes. Here is one way to solve the problem algebraically.

Let x represent the mass of a heart box.
Let y represent the mass of a sun box.
Let z represent the mass of a moon box.

From the information in the diagrams, we can write the following equations:

$$x = 2 \tag{1}$$
$$y + z = 24 \tag{2}$$
$$y + z = 2x + z \tag{3}$$

From equation (1), we already know that a heart box has a mass of 2 kg. From equations (2) and (3), we notice that the left sides are the same, so the right sides of the equations must be equal to each other. This means we know:

$$2x + z = 24 \tag{4}$$

Now, substituting $x = 2$ into equation (4), we get

$$2(2) + z = 24$$

Subtracting 4 from each side of this equation, we get

$$z = 20$$

Finally, substituting $z = 20$ into equation (2), we get

$$y + 20 = 24$$

Subtracting 20 from each side of this equation, we get

$$y = 4$$

So, a heart box has a mass of 2 kg, a sun box has a mass of 4 kg, and a moon box has a mass of 20 kg.