



Problem of the Week

Problem A and Solution

Old Faithful

Problem

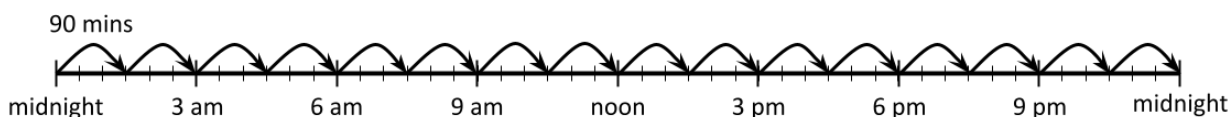
Old Faithful is a geyser in Yellowstone National Park. It is so named because it was believed that it erupted every 60 to 90 minutes all day long. Assuming that Old Faithful erupts at 12 midnight and then erupts every 60 to 90 minutes after the last eruption, answer the following questions.

- (a) After the first eruption at 12 midnight, what is the minimum number of eruptions you could see until up to and including 12 midnight the next night?
- (b) After the first eruption at 12 midnight, what is the maximum number of eruptions you could see until up to and including 12 midnight the next night?

Solution

- (a) One way to solve this problem is to make a timeline.

We would see fewer eruptions if the time between eruptions is longer. The longest gap is 90 minutes. Therefore, the fewest number of eruptions will occur if the time between each eruption is 90 minutes.



From this timeline we can count the ending point of each of the arrows in the diagram and see that, after the first eruption at 12 midnight, there would be 16 eruptions if they happened every 90 minutes. Note that the last eruption would be at exactly midnight on the next night.

Alternatively, we might notice that 90 minutes is equal to $1\frac{1}{2}$ hours, and 180 minutes (or two geyser eruption intervals) is equal to 3 hours. Then we can make a table keeping track of how many eruptions take place over time.

| | | | | | | | | |
|----------------------|---|---|---|----|----|----|----|----|
| Eruptions | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 |
| Hours Elapsed | 3 | 6 | 9 | 12 | 15 | 18 | 21 | 24 |

- (b) We would see more eruptions if the time between eruptions is shorter. The shortest amount of time between eruptions is 60 minutes. Since 60 minutes is equal to 1 hour, and there are 24 hours from 12 midnight until 12 midnight the next night, then the maximum number of eruptions we could see is 24 eruptions.