



Problem of the Week

Problem A and Solution

One Room Schoolhouse

Problem

The following list of numbers represents the ages of students in the one-room schoolhouse in Muggleland:

7, 9, 15, 9, 14, 13, 14, 7, 6, 10, 12, 8, 15, 14, 12,
8, 7, 12, 12, 16, 14, 12, 7, 9, 10, 8, 12, 14, 11, 13

- (a) Are the mode and the median of this set of numbers the same?
Is this relationship the same for every set of numbers?
If so, see if you can explain why. If not, give a set of numbers where the relationship is different.
- (b) Are the median and the mean of this set of numbers the same?
Is this relationship the same for every set of numbers?
If so, see if you can explain why. If not, give a set of numbers where the relationship is different.

Mode refers to the most frequently occurring number in a data set. If there is a tie, then we assign more than one number as the modes of the data set.

Median refers to the middle number in a data set after the numbers have been arranged in order. If a data set has an even number of values, then there are two “middle numbers”. In this case we calculate the sum of the two numbers and divide by 2 to get the median of the data set.

Mean refers to the result of calculating the sum of the numbers in the data set and then dividing the sum by the number of values in the data set. This is what is commonly called the *average*.

Solution

- (a) We can start by filling in a tally chart to count how many times each age appears in our data set.

Age	6	7	8	9	10	11	12	13	14	15	16
Tally											

The most frequently occurring value is 12, so the mode is 12.

We can use the tally chart to list the values in order:

6, 7, 7, 7, 7, 8, 8, 8, 9, 9, 9, 10, 10, 11, 12,
12, 12, 12, 12, 12, 13, 13, 14, 14, 14, 14, 14, 15, 15, 16



Since there is an even number of ages in this data set, we need to calculate the median using the two middle numbers. In this case, we calculate the sum of the 15th and 16th numbers to get $12 + 12 = 24$ and then divide by 2 to get $24 \div 2 = 12$.

So, the median of this set of numbers is 12.

For this data set, the mode and the median are the same. However we would not expect that to always be the case. For example, in the data set

$$1, 1, 1, 1, 2, 3, 4, 5, 6$$

the mode is 1, but the median is 2.

- (b) To determine the mean, we could add up the 30 numbers in the data set to find the sum, but another way to calculate this total would be to use our tally chart again. For each age, we calculate the product of the age and the tally:

Age	6	7	8	9	10	11	12	13	14	15	16
Tally	1	4	3	3	2	1	6	2	5	2	1
Product	6	28	24	27	20	11	72	26	70	30	16

Now, instead of adding 30 numbers together, we only need to find the sum of the 11 products to determine the total to be

$$6 + 28 + 24 + 27 + 20 + 11 + 72 + 26 + 70 + 30 + 16 = 330$$

Now, to find the mean we can use skip counting or divide by the number of ages, which is 30, to get $330 \div 30 = 11$.

So, the mean is 11.

For this data set, the mean and the median are different. However, we would not expect that to always be the case. For example, in the data set

$$5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5$$

the mean is 5 and the median is also 5.