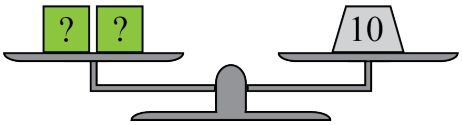
Problem of the Week
Problem B and Solution
A Balancing Act

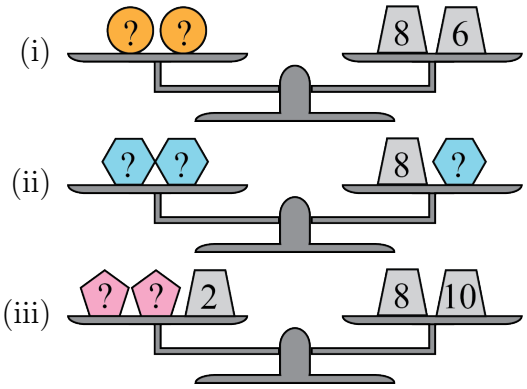
Problem

If a scale is balanced, then the total mass on each side of the scale is the same. Consider the following balanced scale, where the number on an object represents its mass, in grams, and two identical objects with question marks on them have the same unknown mass.

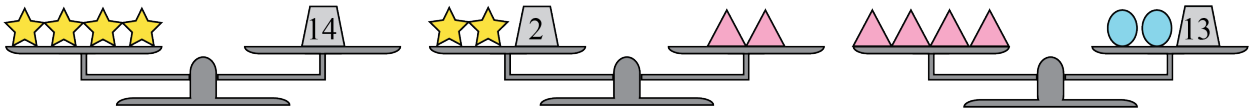


Since the right side has a mass of 10 g, it follows that the two squares must also have a total mass of 10 g. Since the square objects are identical, they must each have a mass of $10 \div 2 = 5$ g.

(a) Find the mass of the indicated shape for each of the three balanced scales.



(b) Using the same idea as in part (a), determine the mass of each symbol in the balanced scales shown. Note that here, the information from the previous scale is used in solving the next one.



Solution

(a) (i) Since 2 circles have a mass of $8 + 6 = 14$ g, it follows that one circle has a mass of $14 \div 2 = 7$ g.

(ii) If we remove one hexagon from each side of the scale, the scale will remain balanced because the hexagons have equal mass. Then we see that one hexagon has a mass of 8 g.

(iii) If we remove 2 g from each side of the scale, then it follows that 2 pentagons have a mass of $8 + 10 - 2 = 16$ g. Then one pentagon has a mass of $16 \div 2 = 8$ g.

(b) From the first scale, we see that 4 stars have a mass of 14 g, so it follows that one star has a mass of $14 \div 4 = 7 \div 2 = 3\frac{1}{2}$ g.

In the second scale there are two stars and an object with a mass of 2 g on the left side. These have a total mass of $3\frac{1}{2} + 3\frac{1}{2} + 2 = 9$ g. Then, two triangles have a mass of 9 g, so one triangle has a mass of $9 \div 2 = 4\frac{1}{2}$ g.

In the third scale there are four triangles on the left side. These have a total mass of $4 \times 4\frac{1}{2} = 18$ g. If we subtract 13 g from each side of this scale, then each side will have a mass of $18 - 13 = 5$ g. Thus, 2 ovals have a mass of 5 g, so one oval has a mass of $5 \div 2 = 2\frac{1}{2}$ g.