



Problem of the Week

Problem B and Solution

A String of Beads

Problem

Aurora is making a beaded necklace using black and green beads. The black beads are all 1.2 cm wide and the green beads are all 4 mm wide. Aurora will make her necklace by alternating the black and green beads.

- If Aurora wants her necklace to be 80 cm long, how many beads will she need in total?
- If the black beads cost \$0.10 each and the green beads cost \$0.03 each, how much will it cost for Aurora to buy all the beads she needs for her necklace?
- Would it cost more or less for Aurora to buy the beads if instead of alternating the black and green beads, she put two green beads after each black bead? Explain.

Solution

- First we need to write the widths of the beads with the same unit of measurement. If we choose centimetres, then the green beads are $4 \div 10 = 0.4$ cm wide. Since Aurora is alternating black and green beads, the necklace will be made up of pairs of black and green beads. Each pair of black and green beads is $1.2 + 0.4 = 1.6$ cm wide. We need to determine how many pairs of black and green beads will fit on the necklace. Since $80 \div 1.6 = 50$, there will be 50 pairs of black and green beads on the necklace. So Aurora will need 50 black beads and 50 green beads, which is a total of 100 beads.
- Aurora needs 50 black beads. Since the black beads cost \$0.10 each, it will cost $50 \times \$0.10 = \5 to buy them all. Aurora needs 50 green beads. Since the green beads cost \$0.03 each, it will cost $50 \times \$0.03 = \1.50 to buy them all. Therefore, in total, it will cost $\$5 + \$1.50 = \$6.50$ to buy all the beads for the necklace.
- If Aurora puts two green beads after each black bead, then the necklace will be made up of groups of one black bead and two green beads. Each of these groups is $1.2 + 0.4 + 0.4 = 2$ cm wide. Since the necklace is 80 cm long, and $80 \div 2 = 40$, it follows that 40 of these groups will fit on the necklace. So the necklace will have 40 black beads and $40 \times 2 = 80$ green beads. Since the black beads cost \$0.10 each, it will cost $40 \times \$0.10 = \4 to buy them all. Since the green beads cost \$0.03 each, it will cost $80 \times \$0.03 = \2.40 to buy them all. Therefore, in total, it will cost $\$4 + \$2.40 = \$6.40$ to buy all the beads for the necklace. Since $\$6.40 < \6.50 , it will be cheaper to buy the beads if Aurora puts two green beads after each black bead.

Alternatively, we could have justified this without doing all the calculations. Notice that the width of three green beads is $3 \times 0.4 = 1.2$ cm, which is the width of one black bead. However, the cost of three green beads is $3 \times \$0.03 = \0.09 , but the cost of one black bead is \$0.10. So three green beads take up the same space as one black bead, but are \$0.01 cheaper to buy. If Aurora puts two green beads after each black bead instead of alternating the black and green beads, then she will end up using more green beads and fewer black beads in her necklace. Every time she replaces one black bead with three green beads she will save \$0.01, so it will be cheaper to buy the beads if Aurora puts two green beads after each black bead.