Problem of the Week
Problem C and Solution
Gone Shopping

Problem

While grocery shopping, Terry has a way to approximate the total cost of his purchases. He simply approximates that each item will cost \$3.00.

One day, Terry purchased 20 items. He purchased items that each had an actual cost of either \$1.00, \$3.00, or \$7.50. Exactly seven of the purchased items had an actual cost of \$3.00. If the total actual cost of the 20 items was the same as the total approximated cost, how many items had an actual cost of \$7.50?

Solution

The total approximated cost for the 20 items is $20 \times \$3 = \$60$. Since the total actual cost is the same as the total approximated cost, the total actual cost for the 20 items is \$60. Since 7 of the items cost \$3.00, it cost Terry $7 \times \$3 = \$21$ to buy these items. Therefore, the remaining $20 - 7 = 13$ items cost $\$60 - \$21 = \$39$.

From this point, we will continue with two different solutions.

Solution 1

In this solution, we will use systematic trial-and-error to solve the problem.

Let s represent the number of items Terry bought with an actual cost of \$7.50 and d represent the number of items that Terry bought with an actual cost \$1.00. Then the total cost of the \$7.50 items would be $7.5s$. Also, the total cost of the \$1.00 items would be $1d = d$. Since Terry's total remaining cost was \$39, then $7.5s + d = 39$. We also know that $s + d = 13$.

At this point we can systematically pick values for s and d that add to 13 and substitute into the equation $7.5s + d = 39$ to find the combination that works. (We can observe that $s < 6$ since $7.5 \times 6 = 45 > 39$. If this were the case, then d would have to be a negative number.)

Let's start with $s = 3$. Then $d = 13 - 3 = 10$. The cost of these items would be $7.5 \times 3 + 10 = 22.50 + 10 = \$32.50$, which is less than \$39.

So let's try $s = 4$. Then $d = 13 - 4 = 9$. The cost of these items would be $7.5 \times 4 + 9 = 30 + 9 = \$39$, which is the amount we want.

Therefore, Terry purchased 4 items that cost \$7.50.

Solution 2

In this solution, we will use algebra to solve the problem.

Let s represent the number of items that cost \$7.50. Therefore, $(13 - s)$ represents the number of items that cost \$1.00. Also, the total cost of the \$7.50 items would be $7.5s$, the total cost of the \$1.00 items would be $1 \times (13 - s) = 13 - s$, and the total of these two is $7.5s + 13 - s = 6.5s + 13$.

Since Terry's total remaining cost was \$39.00, we must have

$$\begin{aligned} 6.5s + 13 &= 39 \\ 6.5s + 13 - 13 &= 39 - 13 \\ 6.5s &= 26 \\ \frac{6.5s}{6.5} &= \frac{26}{6.5} \\ s &= 4 \end{aligned}$$

Therefore, Terry purchased 4 items that cost \$7.50.