



## Problem of the Week

### Problem C and Solution

### Two Squares

#### Problem

Simone has a rope that is 60 cm long. They cut the rope into two pieces so that the ratio of the lengths of the two pieces is 7 : 3. Each piece of the rope is then arranged, with its two ends touching, to form a square.

What is the total area of the two squares?

#### Solution

Since the rope is cut in the ratio of 7 : 3, the ratio of the longer piece to the whole rope will be 7 : (7 + 3) or 7 : 10. This means the length of the longer piece will be  $\frac{7}{10}$  of the length of the whole rope. Similarly, the length of the shorter piece would be  $\frac{3}{10}$  of the length of the whole rope. Therefore, the longer piece is  $\frac{7}{10}$  of 60 or  $\frac{7}{10} \times 60 = 42$  cm long. Also, the shorter piece is  $\frac{3}{10}$  of 60 or  $\frac{3}{10} \times 60 = 18$  cm long.

Each of the two pieces is then used to form a square. The perimeter of each square is the length of the rope used to form it. The side length of the longer square is  $42 \div 4 = 10.5$  cm and the side length of the shorter square is  $18 \div 4 = 4.5$  cm.

To find the area of each square, we multiply the length by the width. In effect, to find the area of the square, we square the side length. Thus, the area of the larger square is  $10.5 \times 10.5 = 10.5^2 = 110.25$  cm<sup>2</sup> and the area of the smaller square is  $4.5 \times 4.5 = 4.5^2 = 20.25$  cm<sup>2</sup>.

Therefore, the total area of the two squares is  $110.25 + 20.25 = 130.5$  cm<sup>2</sup>.

#### FOR FURTHER THOUGHT:

The ratio of the area of the larger square to the area of the smaller square is

$$110.25 : 20.25 = 11025 : 2025 = 441 : 81 = 49 : 9 = 7^2 : 3^2$$

Notice that the ratio of the perimeter of the larger square to the perimeter of the smaller square is 7 : 3 and the ratio of their areas is  $7^2 : 3^2$ . In general, if the ratio of the perimeters of two squares is  $a : b$ , is it true that the ratio of the areas of the two squares is  $a^2 : b^2$ ?