



Problem of the Week

Problem C and Solution

Wacky Race

Problem

Four students participate as a team in a 1000 m wacky relay race. In a wacky relay race, the students each run a portion of the 1000 m length, but they do not run equal lengths. Andrea and Billie run $\frac{1}{8}$ and $\frac{1}{5}$ of the total length, respectively. Carol runs the average of what Andrea and Billie run. Dana runs the remainder of the length.

Determine the fraction of the total length that Dana runs.

Solution

In the first solution, we solve the problem by working with the fractions and without calculating the length that each person runs. In the second solution, we determine the fraction of the length that Dana runs by calculating the distance Dana runs and dividing by the total length of the race.

Solution 1

Carol runs the average of $\frac{1}{8}$ and $\frac{1}{5}$ of the total length of the race.

Therefore, Carol runs $\frac{\frac{1}{8} + \frac{1}{5}}{2} = \frac{\frac{5}{40} + \frac{8}{40}}{2} = \frac{\frac{13}{40}}{2} = \frac{13}{80}$ of the race.

Dana runs the remainder of the race.

Therefore, Dana runs $1 - \frac{1}{8} - \frac{1}{5} - \frac{13}{80} = \frac{80}{80} - \frac{10}{80} - \frac{16}{80} - \frac{13}{80} = \frac{41}{80}$ of the race. Dana runs just over half of the race.

Solution 2

Andrea runs $\frac{1}{8}$ of the race, so she runs $\frac{1}{8} \times 1000 = 125$ m.

Billie runs $\frac{1}{5}$ of the race, so they run $\frac{1}{5} \times 1000 = 200$ m.

Carol runs the average of what Andrea and Billie run.

Therefore, Carol runs $\frac{125+200}{2} = \frac{325}{2} = 162.5$ m.

Dana runs the remainder of the race.

Therefore, Dana runs $1000 - 125 - 200 - 162.50 = 512.5$ m.

That is, Dana runs $\frac{512.5}{1000} = \frac{5125}{10000} = \frac{41}{80}$ of the race.