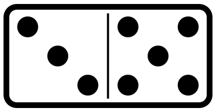


Problem of the Week

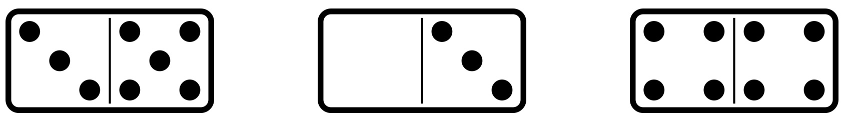
Problem C and Solution

Domi Knows

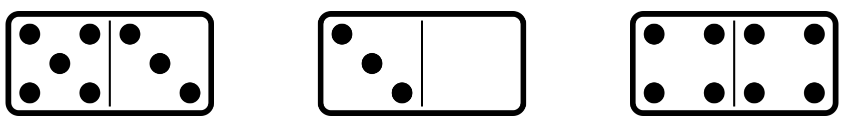
Problem

A domino tile is a rectangular tile with a line dividing its face into two square ends. Each end is marked with a number of dots (also called pips) or is blank.

The first domino shown below is a $[3,5]$ domino, since there are 3 pips on its left end and 5 pips on its right end. The second domino shown below is a $[0,3]$ domino, since there are 0 pips on its left end and 3 pips on its right end. The third domino shown below is a $[4,4]$ domino, since there are 4 pips on its left end and 4 pips on its right end.



We can also rotate the domino tiles. The first domino shown below is a $[5,3]$ domino, since there are 5 pips on its left end and 3 pips on its right end. However, since this tile can be obtained by rotating the $[3,5]$ tile, $[5,3]$ and $[3,5]$ represent the same domino. Similarly, the second domino shown below is a $[3,0]$ domino. Again, note that $[3,0]$ and $[0,3]$ represent the same domino.



A 2-set of dominoes contains all possible tiles with the number of pips on any end ranging from 0 to 2, with no two dominoes being the same. A 2-set of dominoes has the following six tiles: $[0,0]$, $[0,1]$, $[0,2]$, $[1,1]$, $[1,2]$, $[2,2]$. Notice that the three dominoes $[1,0]$, $[2,0]$, and $[2,1]$ are not listed because they are the same as the three dominoes $[0,1]$, $[0,2]$, and $[1,2]$.

Similarly, a 12-set of dominoes contains all possible tiles with the number of pips on any end ranging from 0 to 12, with no two dominoes being the same.

Domi purchased a 12-set of dominoes. How many tiles are in the set?

Solution

Since rotating a domino tile does not change the domino, we will orient each domino so that the smaller number is always on the left end of the domino. Then, for each possible number on the left end of the domino, we will examine the possible numbers that can occur on the right end of the domino, and thus how many dominoes in the set have that number on the left end. We compile this information in a table.

Number on Left End of Domino	Possible Numbers on Right End of Domino	Total Number of Dominoes
0	0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12	13
1	1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12	12
2	2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12	11
3	3, 4, 5, 6, 7, 8, 9, 10, 11, 12	10
4	4, 5, 6, 7, 8, 9, 10, 11, 12	9
5	5, 6, 7, 8, 9, 10, 11, 12	8
6	6, 7, 8, 9, 10, 11, 12	7
7	7, 8, 9, 10, 11, 12	6
8	8, 9, 10, 11, 12	5
9	9, 10, 11, 12	4
10	10, 11, 12	3
11	11, 12	2
12	12	1

Therefore, the total number of dominoes in a 12-set is

$$1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + 11 + 12 + 13 = 91$$

Did you know?

A quick way to calculate the sum

$$1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + 11 + 12 + 13$$

is as

$$\frac{(13)(13+1)}{2}$$

That is, $1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + 11 + 12 + 13 = \dfrac{(13)(13+1)}{2}$.

Can you convince yourself that this is true?

In general, it can be shown that if n is a positive integer, then the sum of the integers from 1 to n is equal to $\dfrac{n \times (n+1)}{2}$.

In other words,

$$1 + 2 + 3 + \ldots + n = \frac{n(n+1)}{2}$$