



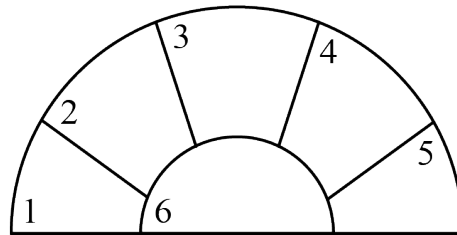
Problem of the Week

Problem D and Solution

Adding Some Colour 2

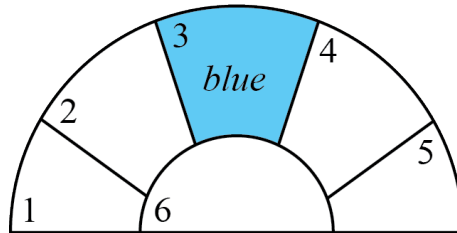
Problem

Finn and Vidya play a game where they take turns colouring regions in the diagram shown red or blue. On their turn, each player colours a region in the diagram that is not bordering another region of the same colour.



After some number of turns, it won't be possible to colour any more regions, and the game will be over. The winner is the player who coloured the last region.

Finn went first. On his turn, he coloured region 3 blue, so after his turn the diagram is coloured as follows.



It is now Vidya's turn and there are five remaining regions. Determine all possibilities for the colour Vidya should use and the region she should choose in order to guarantee that she wins the game, regardless of what Finn does on his remaining turns.

Solution

If Vidya colours region 6 red on her first turn, then she will be guaranteed to win the game, regardless of what Finn does on his remaining turns. First we will show why this is true, and then we will show why all the other possible moves will not guarantee a win for Vidya.

If Vidya colours region 6 red, then the only possible remaining moves are to colour region 1 blue or to colour region 5 blue. Since these moves don't affect each other, Finn will colour one of these regions and Vidya will colour the other and win the game.



The other possible moves for Vidya are to colour region 1 or 5 blue, or to colour region 1, 2, 4, or 5 red.

- If Vidya coloured region 1 blue, then Finn could colour region 4 red. Then the only possible remaining moves would be to colour region 2 red or to colour region 5 blue. Since these moves don't affect each other, Vidya would colour one of these regions and Finn would colour the other and win the game.
- If Vidya coloured region 5 blue, then Finn could colour region 2 red. Then the only possible remaining moves would be to colour region 4 red or to colour region 1 blue. Since these moves don't affect each other, Vidya would colour one of these regions and Finn would colour the other and win the game.
- If Vidya coloured region 1 red, then Finn could colour region 5 red and win the game.
- If Vidya coloured region 5 red, then Finn could colour region 1 red and win the game.
- If Vidya coloured region 2 red, then Finn could colour region 5 blue. Then the only possible remaining moves would be to colour region 4 red or to colour region 1 blue. Since these moves don't affect each other, Vidya would colour one of these regions and Finn would colour the other and win the game.
- If Vidya coloured region 4 red, then Finn could colour region 1 blue. Then the only possible remaining moves would be to colour region 2 red or to colour region 5 blue. Since these moves don't affect each other, Vidya would colour one of these regions and Finn would colour the other and win the game.

Therefore, colouring region 6 red is the only move Vidya can do in order to guarantee that she wins the game, regardless of what Finn does on his remaining turns.