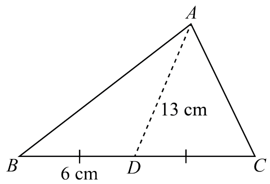


Problem of the Week
Problem D and Solution
There are Two Sides

Problem

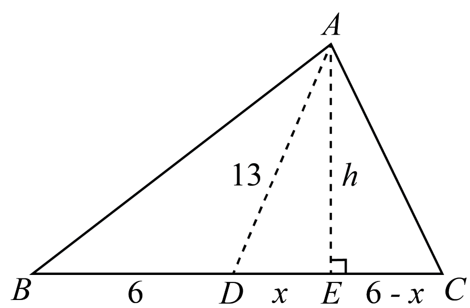
A *median* is a line segment drawn from the vertex of a triangle to the midpoint of the opposite side.

In $\triangle ABC$, a median is drawn from vertex A, meeting side BC at point D. The length of BD is 6 cm and the length of the median AD is 13 cm.

The area of $\triangle ABC$ is 72 cm^2. Determine the lengths of sides AB and AC.

Solution

First we will draw the altitude from vertex A, meeting side BC at point E. Let h be the length of the altitude AE. Let x be the length of DE. Since AD is a median, $DC = BD = 6$. Since E is on DC and the length of DE is x, the length of EC is $6 - x$.



We know the area of $\triangle ABC$ is 72 cm^2. Also, since $BD = DC = 6$ cm, it follows that $BC = 12$ cm. Thus,

$$\begin{aligned} \frac{BC \times AE}{2} &= 72 \\ \frac{12h}{2} &= 72 \\ h &= 12 \end{aligned}$$

Since $\triangle AED$ is right-angled, we can use the Pythagorean Theorem as follows.

$$\begin{aligned} DE^2 + AE^2 &= AD^2 \\ x^2 + 12^2 &= 13^2 \\ x^2 &= 13^2 - 12^2 \\ x^2 &= 169 - 144 = 25 \end{aligned}$$

Since $x > 0$, it follows that $x = 5$ cm. Thus, $BE = 6 + x = 6 + 5 = 11$ cm, and $EC = 6 - x = 6 - 5 = 1$ cm.

Since $\triangle AEB$ is right-angled, we can use the Pythagorean Theorem as follows.

$$\begin{aligned} AE^2 + BE^2 &= AB^2 \\ 12^2 + 11^2 &= AB^2 \\ AB^2 &= 144 + 121 = 265 \end{aligned}$$

Since $AB > 0$, it follows that $AB = \sqrt{265}$ cm.

Since $\triangle AEC$ is right-angled, we can use the Pythagorean Theorem as follows.

$$\begin{aligned} AE^2 + EC^2 &= AC^2 \\ 12^2 + 1^2 &= AC^2 \\ AC^2 &= 144 + 1 = 145 \end{aligned}$$

Since $AC > 0$, it follows that $AC = \sqrt{145}$ cm.

Therefore, the lengths of sides AB and AC are $\sqrt{265}$ cm and $\sqrt{145}$ cm, respectively. These are approximately equal to 16.3 cm and 12.0 cm.