



Problem of the Week

Problem D and Solution

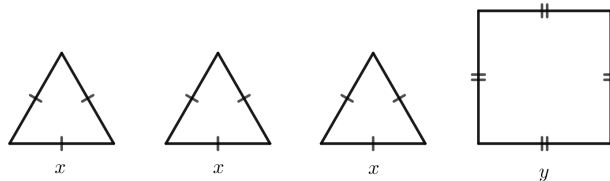
Three Triangles and a Square

Problem

Simon has a rope that is 200 cm long. They cut the rope into four pieces so that one piece can be arranged, with its two ends touching, to form a square, and the three remaining pieces can be arranged, with each having its two ends touching, to form three identical equilateral triangles. If all four shapes have integer side lengths, in cm, determine all possibilities for the side lengths of each triangle and the square.

Solution

Let x represent the side length, in cm, of each equilateral triangle and let y represent the side length, in cm, of the square.



The perimeter of each figure is the length of the piece of rope used to form it. For each triangle, the length of rope is $3x$ and for the square the length of rope is $4y$. The total rope used is $3(3x) + 4y = 9x + 4y$. But the length of the rope is 200 cm. Therefore,

$$\begin{aligned} 9x + 4y &= 200 \\ 9x &= 200 - 4y \\ x &= \frac{4(50 - y)}{9} \end{aligned}$$

We are given that x and y are integers, and the information given in the problem implies x and y must be positive. Since both x and y are integers, $4(50 - y)$ must be a multiple of 9. But 4 is not divisible by 9, so $50 - y$ must be divisible by 9. There are five positive multiples of 9 between 0 and 50, namely 9, 18, 27, 36, and 45. So $50 - y$ must be equal to 9, 18, 27, 36, or 45. It follows that y is equal to 41, 32, 23, 14, or 5. The corresponding values of x are computed in the table below.

y	$4y$	$200 - 4y$	$x = \frac{200-4y}{9}$
41	164	36	4
32	128	72	8
23	92	108	12
14	56	144	16
5	20	180	20

Thus, there are 5 possibilities. When the side length of the square is 41 cm, the side length of each triangle is 4 cm; when the side length of the square is 32 cm, the side length of each triangle is 8 cm; when the side length of the square is 23 cm, the side length of each triangle is 12 cm; when the side length of the square is 14 cm, the side length of each triangle is 16 cm; and when the side length of the square is 5 cm, the side length of each triangle is 20 cm.