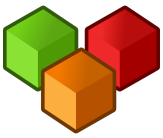

Problem of the Week
Problem D and Solution
Caen's Cubes

Problem

Caen has a cube with a volume of n cm^3. They cut this cube into n smaller cubes, each with a side length of 1 cm. The total surface area of the n smaller cubes is ten times the surface area of Caen's original cube. Determine the side length of Caen's original cube.

Solution

Let the side length of Caen's original cube be x cm, where $x > 0$. It follows that $n = x^3$.

Each of the six sides of Caen's original cube has area x^2 cm^2, so the total surface area of the original cube is $6x^2$ cm^2.

Consider one of the smaller cubes. The area of one the six faces is 1 cm^2. So, the surface area of one of these smaller cubes is 6 cm^2. Thus, the total surface area of the n smaller cubes is $6n$ cm^2.

Since the total surface area of the n cubes is ten times the surface area of Caen's original cube, we have

$$6n = 10(6x^2)$$

Dividing both sides by 6, we have

$$n = 10x^2$$

But $n = x^3$, so this tells us that

$$x^3 = 10x^2$$

Since $x > 0$, we have $x^2 > 0$. Dividing both sides by x^2, we find that $x = 10$.

Therefore, the side length of Caen's original cube was 10 cm.

Extension:

If the combined surface area of the n cubes with a side length of 1 cm was Q times the surface area of the original uncut cube, then the side length of the original uncut cube would have been Q cm. Can you see why?