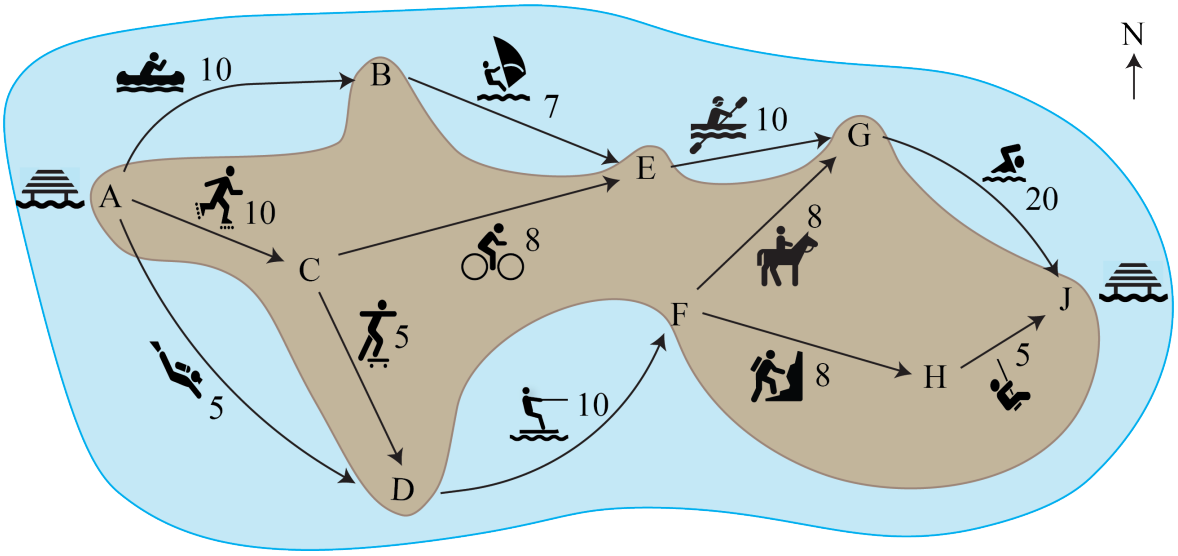
Problem of the Week
Problem E and Solution
Adventure Travel

Problem

A tour company is planning adventure day trips to a small island. Every morning, a boat will take a group of people to the dock on the west side of the island. Each person will then choose a route to travel through the island, doing different activities along the way. The final activities will finish at the dock on the east side of the island, where a boat will take everyone back to the mainland in the evening.

The map below shows all the possible routes people can travel through the island starting from the dock on the west side (A) and finishing at the dock on the east side (J). The activity for each section is shown, as well as the maximum number of people that can do each activity per day due to equipment and/or time constraints.



What is the maximum number of people that can travel from A to J in one day using only the routes and activities shown?

This problem was inspired by a past Beaver Computing Challenge (BCC) problem.

Solution

The maximum number of people that can travel from A to J in one day using only the routes and activities shown is 20. First we will show a possible way that 20 people can travel from A to J, and then we will prove that this is the maximum.

After the 20 people arrive at A, 5 of them should go to B, 10 of them should go to C, and 5 of them should go to D. The people will then be distributed as follows.

Location	A	B	C	D	E	F	G	H	J
Number of People	0	5	10	5	0	0	0	0	0

From B, the only possible route goes to E, so everyone from B should go to E. From C, 5 of the people should go to E and the remaining 5 people should go to D. The people will then be distributed as follows.

Location	A	B	C	D	E	F	G	H	J
Number of People	0	0	0	10	10	0	0	0	0

From D, the only possible route goes to F. Similarly from E, the only possible route goes to G. So all of the people at D should go to F and all of the people at E should go to G. The people will then be distributed as follows.

Location	A	B	C	D	E	F	G	H	J
Number of People	0	0	0	0	0	10	10	0	0

From F, 5 of the people should go to G and the remaining 5 people should go to H. The people will then be distributed as follows.

Location	A	B	C	D	E	F	G	H	J
Number of People	0	0	0	0	0	0	15	5	0

Everyone from G and H should then go to J. Using these routes, a total of 20 people can travel from A to J. Note that this is not the only possible way for 20 people to travel from A to J.

We now must show that it is not possible for more than 20 people to travel from A to J in one day. Suppose we separate the island into two groups. The west group contains A, B, C, D, and E. The east group contains F, G, H, and J. In order to travel from A to J, people must travel from the west group to the east group. However there are only two ways to travel between the west group and the east group. At most 10 people can travel from E to G, and at most 10 people can travel from D to F in one day. So, in total, at most $10 + 10 = 20$ people can travel from the west group to the east group in one day. Since we have found a way for 20 people to travel from A to J in one day, it follows that the maximum number of people that can travel from A to J in one day is 20.