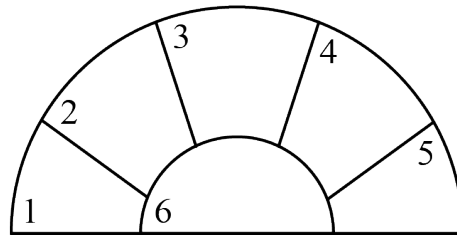# Problem of the Week
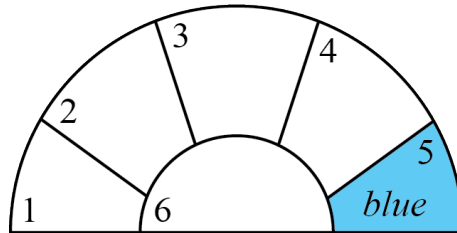## Problem E and Solution
Adding Some Colour 3

Problem

Lucia and Henrik play a game where they take turns colouring regions in the diagram shown red or blue. On their turn, each player colours a region in the diagram that is not bordering another region of the same colour.

After some number of turns, it won't be possible to colour any more regions, and the game will be over. The winner is the player who coloured the last region.

Lucia went first. On her turn, she coloured region 5 blue, so after her turn the diagram is coloured as follows.

It is now Henrik's turn and there are five remaining regions. Determine all possibilities for the colour Henrik should use and the region he should choose in order to guarantee that he wins the game, regardless of what Lucia does on her remaining turns.

Solution

If Henrik colours region 1 red on his first turn, then he will be guaranteed to win the game, regardless of what Lucia does on her remaining turns. First we will show why this is true, and then we will show why all the other possible moves will not guarantee a win for Henrik.

If Henrik colours region 1 red, then the only possible moves for Lucia are to colour region 2 blue, region 3 red or blue, or region 4 red.

- If Lucia colours region 2 blue, then the only possible moves for Henrik are to colour region 3 or 4 red. After Henrik chooses one of these moves, there will be no possible moves left and Henrik will win the game.

- If Lucia colours region 3 blue, then the only possible move for Henrik is to colour region 4 red. After Henrik does this, there will be no possible moves left and Henrik will win the game.

- If Lucia colours region 3 red, then the only possible move for Henrik is to colour region 2 blue. After Henrik does this, there will be no possible moves left and Henrik will win the game.

- If Lucia colours region 4 red, then the only possible moves for Henrik are to colour region 2 or region 3 blue. After Henrik chooses one of these moves, there will be no possible moves left and Henrik will win the game.

Thus, if Henrik colours region 1 red, then he is guaranteed to win the game, regardless of what Lucia does on her remaining turns.

The other possible moves for Henrik are to colour region 1, region 2, or region 3 blue, or to colour region 2, region 3, region 4, or region 6 red.

- If Henrik coloured region 1 blue, then Lucia could colour region 3 red. There would then be no possible moves left, so Lucia would win the game.

- If Henrik coloured region 2 blue, then Lucia could colour region 6 red. There would then be no possible moves left, so Lucia would win the game.

- If Henrik coloured region 3 blue, then Lucia could colour region 2 red. Then the only possible remaining moves would be to colour region 1 blue or to colour region 4 red. Since these moves don't affect each other, Henrik would colour one of these regions and Lucia would colour the other and win the game.

- If Henrik coloured region 2 red, then Lucia could colour region 3 blue. Then the only possible remaining moves would be to colour region 1 blue or to colour region 4 red. Since these moves don't affect each other, Henrik would colour one of these regions and Lucia would colour the other and win the game.

- If Henrik coloured region 3 red, then Lucia could colour region 1 blue. There would then be no possible moves left, so Lucia would win the game.

- If Henrik coloured region 4 red, then Lucia could colour region 2 red. Then the only possible remaining moves would be to colour region 1 blue or to colour region 3 blue. Since these moves don't affect each other, Henrik would colour one of these regions and Lucia would colour the other and win the game.

- If Henrik coloured region 6 red, then Lucia could colour region 2 blue. There would then be no possible moves left, so Lucia would win the game.

Therefore, colouring region 1 red is the only move Henrik can do in order to guarantee that he wins the game, regardless of what Lucia does on her remaining turns.