



Problem of the Week

Problem E and Solution

Three Items Only

Problem

Three-Item-Menu is a restaurant that sells only hamburgers, french fries, and soft drinks.

One day, exactly 120 customers made a purchase. Half of the customers purchased at least a hamburger, $\frac{1}{4}$ of the customers purchased at least french fries, but $\frac{1}{3}$ of the customers purchased only a soft drink. Of the customers who bought a hamburger, $\frac{4}{5}$ of them bought at least one other item.

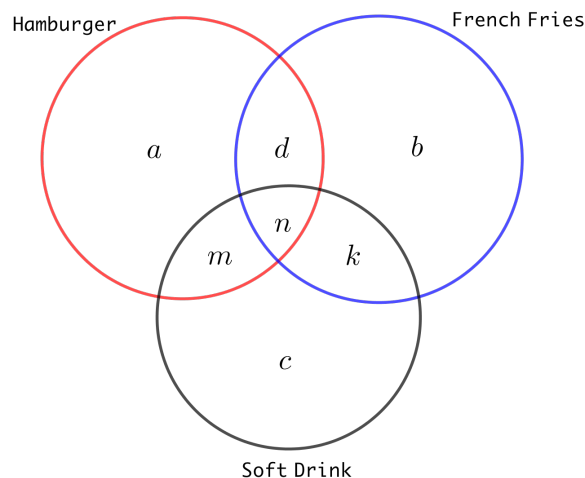
How many customers purchased a hamburger and soft drink but not french fries?

Solution

We start by defining seven variables as follows:

- Let a be the number of customers who purchased a hamburger only.
- Let b be the number of customers who purchased french fries only.
- Let c be the number of customers who purchased a soft drink only.
- Let d be the number of customers who purchased a hamburger and french fries but not a soft drink.
- Let k be the number of customers who purchased french fries and a soft drink but not a hamburger.
- Let m be the number of customers who purchased a hamburger and a soft drink but not french fries.
- Let n be the number of customers who purchased a hamburger, french fries and a soft drink.

We have added these variables to the Venn diagram below. We need to determine the value of m .





We know that 120 customers made a purchase, so

$$a + b + c + d + k + m + n = 120 \quad (1)$$

We are given that half of the customers purchased a hamburger, so $\frac{1}{2} \times 120 = 60$ customers purchased a hamburger. These customers may have also purchased french fries or a soft drink or both or neither. This tells us that

$$a + d + m + n = 60 \quad (2)$$

We are given that $\frac{1}{4}$ of the customers purchased french fries, so $\frac{1}{4} \times 120 = 30$ customers purchased french fries. These customers may have also purchased a hamburger or a soft drink or both or neither. This tells us that

$$b + d + k + n = 30 \quad (3)$$

We are given that $\frac{1}{3}$ of the customers purchased only a soft drink, so $\frac{1}{3} \times 120 = 40$ customers purchased only a soft drink. Therefore, $c = 40$.

We are given that, of the customers who bought a hamburger, $\frac{4}{5}$ bought at least one other item. So, $\frac{4}{5}$ of the 60 customers who purchased a hamburger bought at least one other item. In other words, $\frac{4}{5} \times 60 = 48$ also bought french fries, a soft drink or both. Therefore,

$$d + m + n = 48 \quad (4)$$

Subtracting equation (4) from equation (2), we see that $a = 12$.

Substituting $a = 12$ and $c = 40$ into equation (1), we have $12 + b + 40 + d + k + m + n = 120$, and thus $b + d + k + m + n = 68$, or $(b + d + k + n) + m = 68$. From equation (3) we know $b + d + k + n = 30$, and so $30 + m = 68$, thus $m = 38$. Therefore, 38 customers purchased a hamburger and soft drink but not french fries.

We have determined what was required and can stop here. We do not need to solve for the remaining variables. It actually turns out that in this problem there is not enough information given to determine the values of all of the remaining variables.