



## Problem of the Week

### Problem E and Solution

#### How Far From Here to There



#### Problem

A cyclist leaves the town of Alphaville and heads toward Betaville. She travels at a constant speed of 14 km/h.

At the same time, a jogger and a walker leave Betaville and head toward Alphaville. The walker travels at a constant speed of 6 km/h and the jogger travels at a constant speed of 10 km/h.

If the cyclist passes the walker 4 minutes after passing the jogger, how far apart are the towns Alphaville and Betaville?

#### Solution

Let  $d$  be the distance, in km, between the towns Alphaville and Betaville.

Let  $t$  be the time, in hours, until the jogger and cyclist meet.

Using the formula distance = speed  $\times$  time, in  $t$  hours the cyclist travels  $14t$  km and the jogger travels  $10t$  km.

Between the cyclist and jogger, they travel the total distance between Alphaville and Betaville in  $t$  hours. Therefore,  $d = 14t + 10t = 24t$ .

The cyclist meets the walker 4 minutes, or  $\frac{4}{60} = \frac{1}{15}$  hours, after meeting the jogger. Therefore,  $(t + \frac{1}{15})$  is the time, in hours, until the cyclist meets the walker.

Again, using the formula distance = speed  $\times$  time, in  $(t + \frac{1}{15})$  hours, the cyclist travels  $14(t + \frac{1}{15})$  km and the walker travels  $6(t + \frac{1}{15})$  km.

Between the cyclist and walker, they travel the total distance between Alphaville and Betaville in  $(t + \frac{1}{15})$  hours. Therefore,  $d = 14(t + \frac{1}{15}) + 6(t + \frac{1}{15}) = 20(t + \frac{1}{15})$ .

Thus,  $d = 24t$  and  $d = 20(t + \frac{1}{15})$ . Therefore,

$$24t = 20\left(t + \frac{1}{15}\right)$$

$$24t = 20t + \frac{4}{3}$$

$$4t = \frac{4}{3}$$

$$t = \frac{1}{3}$$

Since  $t = \frac{1}{3}$  hours, we find  $d = 24t = 24\left(\frac{1}{3}\right) = 8$  km.

Therefore, the towns of Alphaville and Betaville are 8 km apart.